Chapter 1

Introduction

In the 1960s, Charnes, Cooper and Kortanek developed the theory of Semi-Infinite Programming (SIP), extending the field of linear and nonlinear programming to a case where infinitely many constraints or infinitely many variables were considered. Basing themselves on Haar's 1924 paper, they defined the notion of "Haar" or Semi-infinite programs which are comprised of the minimization of a linear function of finitely many variables subject to an arbitrary number of linear inequalities. This came to be known as Linear Semi-Infinite Programming (LSIP). These problems were used to explore the borderline between problems that can be solved through algebraic methods and those that require concepts of topology [20]. That same year, they applied LSIP to convex programming where much progress was made by Charnes, Cooper, Kortanek, Dieter, Whinston and Rockafeller. In [27], Rockafeller developed the powerful technique of including perturbations in the convex programming formulation.

Since these first steps, semi-infinite optimization has grown to become an independent research branch. It's theoretic beauty appeals to both pure and applied mathematicians while its wide variety of applications in probablity, statistics, control, and assignment games (see [3], [29], [26], and [28]) attract engineers, management scientists, optimization experts, economists and other researchers from a diverse spectrum of fields. One of the best known applications of semi-infinite programming, Chebyshev approximation, has been the starting point of many important results such as those presented in [27], [13], [7] and [5] to name a few.

Other applications of Linear Semi-Infinite Programming in the areas of risk theory, robotics, statistical design, urban planning and environmental policy making are mentioned in [6] and [15].

Despite its applicability to everyday problems, SIP requires an extensive theoretic base. Requiring knowledge of fields in Mathematics such as calculus, functional analysis and convex analysis. One particular characteristic of SIP that complicates its use is that a point $\overline{x} \in \operatorname{bd} F$ may not have any active constraint. In this thesis we investigate the characteristics of active constraints in the Convex Semi-Infinite Programming Case. In [11] the concept of $\gamma-$active constraints is presented for the Linear Semi-Infinite Programming case. This new concept proves to be very useful in preserving some of the properties of conventional active constraints, hence our objective is to extend the concept of $\gamma-$active constraints presented for LSIP in [11] to the Convex SIP case.

This document is divided as follows. In chapter 2, the detailed definition of the problem is presented along with a literature review of applications, active constraints, stability and optimality conditions in SIP. In Chapter 3 we present some of the background knowledge necessary for achieving our purpose. We present definitions and basic properties (omitting their proofs) in the areas of calculus, convex analysis, and optimization. We also revisit the definition and properties of $\gamma-$active constraints in Linear Semi-Infinite Programming presented in [11]. Chapter 4 explores the concept of $\gamma-$active constraints in CSIP based on the use of the subdifferential while Chapter 5 does the same based on the use of a linearization of the Semi-Infinite Inequality system.In Chapter 6 we compare both approaches, stating their pros and cons, their advantages and shortcomings and finally in Chapter 7 we present conclusions on the research done and results found.