CHAPTER V

COMPUTATIONAL EXPERIENCE

To test both proposed methods 160 instances where used, these instances are the same used in (Ceselli & Righini, 2005) and are divided in four classes α, β, γ and Δ; every class contains 40 instances divided in blocks of 10 instances that have 50, 100, 150 and 200 nodes respectively. Each class varies in the number of p that the aforementioned blocks have. For the class α p is 5, 10, 15 and 20. In class β the values of p are higher 12, 25, 37 and 50. This is true again in class γ, with values for p of 16, 33, 50 and 66. Finally the class Δ presents the highest values of p being allocated with 20, 40, 60 and 80. However, a direct comparison of the results obtained in this work to the ones of (Ceselli & Righini, 2005) cannot be performed as the formulation of the authors allows medians to be assigned to other median-nodes and the formulation for the present work does not allow that. The solutions for the instances are obtained with the solver of the optimization software FICO XPRESS, the instances were executed in a computer Lenovo Think Station, with a processor Intel Xeon, 3.1 GHz and 8GB of RAM.

It is important to note that after a series of trial tests it was decided that the best parameters that were going to be used were: a starting value of ρ of 2; it was also decided that T was going to be 30, this means that if after 30 iterations no better value was found for the Lagrangean relaxation then ρ would be halved, becoming 1, then 0.5 and so on. The maximum number of iterations is given by the number K which was set at 1200. Also the number ε is 0.0001, this number is used as a second stopping condition and if the step size ever falls below this number then the whole program is finished despite the number of iteration.

The tables 1, 2, 3 and 4 present the different results obtained for the classes α, β, γ and Δ respectively. Each table is divided in 13 columns. The first 3 columns characterize each instance: the first one states the number p of medians to be located, the second one the number N of nodes and the third one is the number of instance that goes from 1 to 40 for each table. Columns 4 to 8 show the results of the instances when using the Lagrangean heuristic: column 4 indicates the best upper bound found solving the generalized assignment problem; column 5 shows the best lower bound which is the solution obtained when trying to maximize the value of the Lagrangean relaxation; then column 6 presents the relative gap between the upper and lower bound this is obtained by $(UB - LB)/LB * 100$ where UB stands for upper bound and LB for lower bound, the relative gap is used to measure the quality of the results; column 7 is the execution time in seconds required to reach the number of iterations that are shown in column 8. After any block of 10 instances (before p and N change) a row is included and it presents the average gap for that block, this is useful as it can more easily compare the effectiveness of both approaches when solving the same type of problems.

Columns 9 through 13 show the same information when using the cluster median improvement heuristic. Column 9 presents the upper bound, where the CMI is used; column 10 the lower bound obtained through the subgradient method; the relative gap, execution time and number of iterations are reported in columns 11, 12 and 13 respectively.

While the tables 1 to 4 present and allow comparing the results of the whole Lagrangean heuristics and the quality of their results, it would also be interesting to make a comparison and show the quality of just the lower bounds. The results are presented in the tables 5 and 6. Table 5 presents the results for the classes α and β, while table 6 does the same for classes γ and Δ. Each table has 12 columns. The first three columns identify each instance like in the previous tables: they show p, N and the number of instance respectively. Column 4 has the result of the lower bound obtained by the Lagrangean heuristic, column 5 presents the lower bound obtained when using the Lagrangean heuristic plus the cluster median improvement heuristic, finally the column 6 presents the result of the linear relaxation of the problem. As explained before, this is obtained by solving the problem when the integrality constraints are dropped. Columns 7 to 12 present the very same results but for a different class of instances.

After analyzing the results, we could give some general results: the CMI heuristic is better at obtaining upper bounds but not as good when obtaining lower bounds. Also the execution time grows considerably. Now more specific results will be discussed for each table.

The table 1 shows the instances of class α, according to (Ceselli & Righini, 2005) these are the easiest to be solved, and that can be corroborated with the execution times required when compared to instances with the same N that belong to other classes. As it can be noted when comparing the upper bounds from each method, it can be noted that the CMI was able to improve the upper bounds in most instances except for the instances 1, 2, 4, 5 and 11. It is suspected that this is due to the Lagrangean heuristic obtaining the optimal value for the problem and thus is only logic why the CMI cannot provide a better upper bound, but in order to prove this theory solving the problems to optimality must be done. As mentioned before most of times the CMI is not able to obtain better lower bounds when compared to the regular Lagrangean heuristic, despite this for instances 1, 2, 9, 15 and 20 the CMI was able to obtain better lower bounds. Regarding to the gaps, the CMI is able to obtain better gaps the majority of times. It is notorious that the instances of this class, which are considered being the easiest ones to be solved, contain almost all the cases where the CMI obtained worse gaps; the instances are the 4, 5,6,7,11,21 and 30. Even so if we check the average gap for any given block of ten instances we can corroborate that in all cases the CMI presents better results, even if humble for the smaller instances it becomes more notorious in bigger instances.

Table 1. Results of the Lagrangean Heuristic and the CMI Heuristic for the Class α instances.

			Lagrangean Heuristic					**CMI Heuristic**				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
5	**50**	1	713	692.401	2.98%	25.126	505	713	697.505	2.22%	76.396	1200
		2	740	739.914	0.01%	26.814	508	740	739.931	0.01%	41.014	630
		3	753	747.817	0.69%	71.629	1199	751	747.515	0.47%	80.912	1200
		4	651	649.846	0.18%	34.537	646	651	648.95	0.32%	56.833	855
		5	664	654.655	1.43%	31.861	608	664	653.107	1.67%	56.078	862
		6	778	777.999	0.00%	30.828	567	778	772.088	0.77%	79.501	1200
		7	789	778.249	1.38%	79.166	1200	787	776.077	1.41%	91.172	1200
		8	823	769.808	6.91%	67.371	1161	820	769.369	6.58%	59.323	890
		9	730	706.573	3.32%	41.062	741	715	706.9	1.15%	49.171	732
		10	842	802.97	4.86%	59.583	935	829	801.8	3.39%	81.504	1057
		Average gap			**2.18%**					**1.80%**		
10	**100**	11	1006	998.272	0.77%	163.84	1200	1006	985.311	2.10%	220.205	1200
		12	987	957.958	3.03%	144.514	1099	968	946.171	2.31%	236.241	1200
		13	1033	1019.33	1.34%	101.524	743	1026	1017.06	0.88%	237.346	1200
		14	1007	967.944	4.03%	199.249	1200	982	963.742	1.89%	239.834	1200
		15	1094	1061.59	3.05%	164.506	1200	1092	1063.08	2.72%	212.994	1200
		16	1000	935.514	6.89%	159.765	1200	954	931.755	2.39%	204.093	1200
		17	1093	1013.57	7.84%	171.619	1200	1034	1011.06	2.27%	260.919	1200
		18	1059	1028.4	2.98%	168.211	1200	1043	1028.4	1.42%	230.685	1200
		19	1116	1019.13	9.51%	172.826	1200	1031	1000.21	3.08%	238.016	1200
		20	1071	946.983	13.10%	192.791	1200	1005	953.04	5.45%	257.251	1200
		Average gap			**5.25%**					**2.45%**		

Table 1. Continuation

			Lagrangean Heuristic					CMI Heuristic				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
15	**150**	21	1307	1275.16	2.50%	346.198	1200	1296	1255.42	3.23%	530.408	1200
		22	1378	1229.95	12.04%	382.398	1200	1256	1198.78	4.77%	747.616	1200
		23	1337	1265.47	5.65%	468.263	1200	1285	1244.69	3.24%	873.147	1200
		24	1310	1196.2	9.51%	308.741	1200	1223	1181.57	3.51%	560.934	1200
		25	1382	1182.66	16.86%	262.704	1200	1204	1166.1	3.25%	448.655	1200
		26	1310	1247.33	5.02%	298.86	1200	1265	1237.1	2.26%	442.9	1200
		27	1443	1300.1	10.99%	453.836	1200	1326	1280.51	3.55%	1071.73	1200
		28	1375	1226.5	12.11%	276.046	1200	1235	1192.31	3.58%	528.187	1200
		29	1328	1215.27	9.28%	273.757	1200	1228	1194.64	2.79%	445.022	1200
		30	1206	1184.63	1.80%	269.565	1200	1202	1172.17	2.54%	471.356	1200
		Average gap			**8.58%**					**3.27%**		
20	**200**	31	1482	1367.9	8.34%	573.521	1200	1392	1345.19	3.48%	1443.29	1200
		32	1706	1393.61	22.42%	2883.13	1200	1433	1355.65	5.71%	6918.94	1200
		33	1659	1355.63	22.38%	860.104	1200	1375	1321.63	4.04%	2193.78	1200
		34	1478	1364.77	8.30%	1815.37	1200	1388	1346.26	3.10%	4483.79	1200
		35	1623	1419.21	14.36%	814.492	1200	1463	1390.39	5.22%	2048.98	1200
		36	1442	1371.66	5.13%	806.125	1200	1392	1358.95	2.43%	1835.93	1200
		37	1629	1447.67	12.53%	575.248	1200	1477	1418.97	4.09%	1513.56	1200
		38	1513	1365.24	10.82%	1103.16	1200	1388	1336.52	3.85%	2797.89	1200
		39	1425	1355.09	5.16%	591.969	1200	1375	1337.3	2.82%	1553.99	1200
		40	1587	1408.88	12.64%	973.307	1200	1428	1380.82	3.42%	2782.74	1200
		Average gap			**12.21%**					**3.82%**		

Class β results are shown in table 2. The advantages of the CMI are more notorious in this class, as the average gaps were reduced more than three times in the instances 11 to 40, the most difficult ones. The gap is improved in almost all cases, being the exception the instance 7, this is due to the lower bound worsening importantly but even so the upper bound was improved. Also in the cases of the instances 1, 5, 8 and 10 the lower bound was improved. In general, these instances where better to show the performance of the CMI.

Table 2. Results of the Lagrangean Heuristic and the CMI Heuristic for the Class β instances.

			Lagrangean Heuristic					CMI Heuristic				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
12	**50**	1	386	372.313	3.68%	71.205	1183	383	373.175	2.63%	80.084	1190
		2	438	407.985	7.36%	49.843	982	412	407.709	1.05%	77.844	1115
		3	424	398.247	6.47%	73.353	1200	405	397.887	1.79%	93.247	1200
		4	434	364.83	18.96%	86.664	1200	384	363.883	5.53%	92.969	1200
		5	458	415.68	10.18%	86.741	1200	433	415.707	4.16%	108.393	1200
		6	521	464.371	12.19%	81.711	1200	485	463.043	4.74%	93.782	1200
		7	465	422.224	10.13%	86.255	1200	445	381.044	16.78%	30.8	362
		8	458	382.879	19.62%	42.239	651	403	389.119	3.57%	100.946	1200
		9	448	419.578	6.77%	84.364	1200	436	418.998	4.06%	102.954	1200
		10	512	430.328	18.98%	93.242	1200	461	431.758	6.77%	122.225	1200
		Average gap			**11.43%**					**5.11%**		
25	**100**	11	674	526.792	27.94%	197.651	1200	547	513.719	6.48%	440.201	1200
		12	641	494.567	29.61%	201.681	1200	516	488.153	5.70%	442.75	1200
		13	672	533.682	25.92%	207.074	1200	567	532.021	6.57%	348.984	1200
		14	639	529.015	20.79%	223.601	1200	559	526.309	6.21%	517.302	1200
		15	656	567.449	15.61%	190.405	1200	590	562.183	4.95%	366.584	1200
		16	620	517.586	19.79%	220.6	1200	539	513.706	4.92%	395.073	1200
		17	594	533.946	11.25%	245.482	1200	549	527.646	4.05%	550.699	1200
		18	582	499.29	16.57%	194.615	1200	517	494.527	4.54%	329.722	1200
		19	651	530.451	22.73%	197.097	998	561	529.983	5.85%	401.599	1200
		20	657	511.23	28.51%	364.935	1200	561	507.997	10.43%	813.898	1200
		Average gap			**21.87%**					**5.97%**		
37	**150**	21	759	672.951	12.79%	484.477	1200	695	668.897	3.90%	1498.96	1200
		22	842	639.438	31.68%	810.114	1200	669	624.721	7.09%	2420.89	1200
		23	808	637.836	26.68%	812.581	1200	672	616.877	8.94%	2209.12	1200
		24	673	583.936	15.25%	426.452	1200	608	580.621	4.72%	1250.83	1200
		25	800	626.943	27.60%	345.946	1200	649	616.6	5.25%	1148.99	1200
		26	771	643.201	19.87%	439.495	1200	675	635.593	6.20%	1131.2	1200
		27	923	708.168	30.34%	1455.22	1200	758	695.457	8.99%	3713.44	1200
		28	808	632.093	27.83%	447.719	1200	670	627.037	6.85%	1415.4	1200
		29	709	640.069	10.77%	385.438	1200	660	638.29	3.40%	1141.38	1200
		30	741	618.012	19.90%	426.593	1200	640	613.298	4.35%	1100.65	1200
		Average gap			**22.27%**					**5.97%**		

Table 2. Continuation

			Lagrangean Heuristic					CMI Heuristic				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
50	**200**	31	914	709.348	28.85%	1104.91	1200	756	696.301	8.57%	3438.95	1200
		32	1066	774.409	37.65%	6633.41	1200	853	754.658	13.03%	18075.3	1200
		33	877	691.898	26.75%	1313.93	1200	725	682.254	6.27%	4514.03	1200
		34	1112	767.204	44.94%	3247.49	1200	830	740.321	12.11%	11005	1200
		35	965	723.974	33.29%	1662.84	1200	779	706.577	10.25%	4067.19	1200
		36	932	687.222	35.62%	1438.07	1200	735	672.982	9.22%	4490.09	1200
		37	844	743.203	13.56%	926.366	1200	803	739.859	8.53%	2729.33	1200
		38	970	728.249	33.20%	2745.47	1200	796	713.514	11.56%	8192.05	1200
		39	823	709.347	16.02%	944.065	1200	764	705.536	8.29%	2432.09	1200
		40	1007	732.859	37.41%	2168.5	1200	800	710.127	12.66%	6491.44	1200
		Average gap			**30.73%**					**10.05%**		

Ceselli and Righini (2005) mention that class γ is the most difficult one to solve. It actually contains some of the problems that took the longest to be solved. The results are presented in the Table 3. We could say that this is where the CMI demonstrate its potential, it improves upon every upper bound, every gap and even in the lower bound of instances 3, 7, 8 and 10. The average gap improvements range from a third to a fifth of those obtained prior to the use of the CMI, and when checked individually there are cases where the improvement reduces the gap into a tenth.

Table 3. Results of the Lagrangean Heuristic and the CMI Heuristic for the Class γ instances.

			Lagrangean Heuristic					CMI Heuristic				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
16	**50**	1	367	295.264	24.30%	72.41	1200	298	291.239	2.32%	86.813	1200
		2	405	327.992	23.48%	79.023	1200	336	324.7	3.48%	95.578	1200
		3	369	308.931	19.44%	71.504	1163	314	308.992	1.62%	92.982	1160
		4	356	296.067	20.24%	85.146	1200	303	290.503	4.30%	103.386	1200
		5	392	345.44	13.48%	67.38	1040	358	344.729	3.85%	89.732	1108
		6	462	381.416	21.13%	84.462	1200	400	381.376	4.88%	101.778	1200
		7	398	345.626	15.15%	83.333	1200	361	346.677	4.13%	102.656	1200
		8	420	311.581	34.80%	87.242	1200	353	312.999	12.78%	109.388	1200

Table 3. Continuation

			Lagrangean Heuristic					CMI Heuristic				
P	N	Instance	Upper Bound	Lower Bound	Gap	Time	Iterations	Upper Bound	Lower Bound	Gap	Time	Iterations
16	**50**	9	416	362.805	14.66%	89.083	1200	373	358.274	4.11%	110.171	1200
		10	445	365.668	21.70%	98.368	1200	391	365.886	6.86%	142.249	1200
		Average gap			**20.84%**					**4.83%**		
33	**100**	11	573	405.29	41.38%	186.238	1200	422	396.55	6.42%	346.359	1200
		12	556	372.056	49.44%	162.428	1107	408	370.955	9.99%	326.285	1200
		13	583	437.458	33.27%	189.754	1200	478	435.276	9.82%	306.822	1200
		14	647	432.644	49.55%	219.864	1200	469	422.702	10.95%	402.645	1200
		15	619	467.248	32.48%	189.659	1200	490	459.346	6.67%	353.492	1200
		16	548	430.079	27.42%	209.451	1200	466	429.621	8.47%	346.859	1200
		17	583	422.883	37.86%	181.988	1200	443	418.067	5.96%	427.66	1200
		18	586	423.894	38.24%	220.466	1200	467	421.046	10.91%	353.566	1200
		19	610	428.617	42.32%	202.16	1200	465	423.924	9.69%	390.847	1200
		20	573	439.212	30.46%	279.586	1200	474	435.729	8.78%	459.902	1200
		Average gap			**38.24%**					**8.77%**		
50	**150**	21	808	577.772	39.85%	512.713	1200	635	571.517	11.11%	1436.89	1200
		22	788	533.571	47.68%	558.324	1200	594	521.715	13.86%	1814.32	1200
		23	824	543.513	51.61%	660.921	1200	616	525.189	17.29%	2101.88	1200
		24	670	494.854	35.39%	409.935	1200	539	487.53	10.56%	1289.08	1200
		25	634	484.467	30.87%	339.182	1200	525	479.396	9.51%	951.915	1200
		26	712	524.938	35.64%	375.852	1200	575	522.74	10.00%	1102.22	1200
		27	845	562.025	50.35%	708.762	1200	622	547.795	13.55%	2704.25	1200
		28	688	498.73	37.95%	385.518	1200	545	489.759	11.28%	1116.53	1200
		29	661	528.121	25.16%	458.27	1200	566	522.359	8.35%	1070.04	1200
		30	643	486.616	32.14%	366.067	1200	532	480.523	10.71%	1094.65	1200
		Average gap			**38.66%**					**11.62%**		
66	**200**	31	807	569.572	41.69%	638.674	1200	617	561.916	9.80%	2213.61	1200
		32	984	676.471	45.46%	3144.64	1200	785	648.7	21.01%	9935.87	1200
		33	878	595.084	47.54%	1182.94	1200	670	579.776	15.56%	3576.89	1200
		34	950	657.987	44.38%	1606.71	1200	749	636.191	17.73%	4611.91	1200
		35	819	589.528	38.92%	807.553	1200	663	578.133	14.68%	2673.26	1200
		36	821	564.659	45.40%	979.033	1200	638	549.238	16.16%	2876.82	1200
		37	793	615.46	28.85%	638.943	1200	685	612.299	11.87%	2233.12	1200
		38	832	591.989	40.54%	1302.24	1200	663	573.475	15.61%	3976.17	1200
		39	776	572.354	35.58%	616.456	1200	639	565.873	12.92%	1963.09	1200
		40	826	603.277	36.92%	1062.99	1200	683	588.925	15.97%	3420.17	1200
		Average gap			**40.53%**					**15.13%**		

The final class, the Δ has its results reported in table 4. These instances could be considered hard as well. Once again the improvements are in every upper bound, gap and in the lower bound of the instances 5, 7, 9, 10 and 17. Although the improvement of the gaps is not as impressive as in the last class, they are quite good as well reducing the gap from half to a third of the previous one.

As it can be noted when comparing all the upper bounds for both methods, the CMI is able to produce at least equal upper bounds but in 154 out of the 160 cases they are improved and in many cases greatly improved, the bigger improvements come from the harder instances. For the lower bound the results are not as good as in just 18 times out of the 160 the lower bound was improved. But the improvement of the upper bound is mostly way greater than the worsening of the lower bound, as a result in 152 cases the gaps are improved, and once again the better improvements can be seen in harder instances. In the 8 cases where the gap worsens the previous gap was already small enough and the worsening is relatively small, also this only happened in small instances.

We could conclude that the CMI proves to be better for obtaining upper bounds and overall improving the gaps, but this comes with a tradeoff, for every instance it takes more time to get solutions. This is due to the generalized assignment problem having to be solved many more times in the CMI.

Table 4. Results of the Lagrangean Heuristic and the CMI Heuristic for the Class Δ instances.

			Lagrangean Heuristic					**CMI Heuristic**				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
20	**50**	1	364	258.664	40.72%	73.52	1200	275	256.913	7.04%	80.39	1200
		2	372	291.755	27.50%	79.66	1200	311	270.348	15.04%	89.985	1200
		3	400	293.698	36.19%	79.017	1200	322	284.514	13.18%	94.821	1200
		4	352	261.358	34.68%	86.565	1200	281	253.674	10.77%	109.729	1200
		5	443	320.548	38.20%	82.4	1200	365	325.334	12.19%	101.282	1200
		6	471	357.775	31.65%	91.42	1200	370	342.065	8.17%	104.662	1200
		7	456	318.02	43.39%	85.082	1200	367	324.862	12.97%	102.438	1200
		8	462	282.596	63.48%	86.505	1200	313	271.253	15.39%	135.939	1200
		9	479	357.946	33.82%	90.489	1200	413	359.131	15.00%	112.938	1200
		10	533	345.721	54.17%	106.804	1200	474	350.993	35.05%	153.847	1200
		Average			**40.38%**					**14.48%**		

Table 4. Continuation

			Lagrangean Heuristic					CMI Heuristic				
P	**N**	**Instance**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**	**Upper Bound**	**Lower Bound**	**Gap**	**Time**	**Iterations**
40	**100**	11	562	392.2	43.29%	185.657	1200	427	386.834	10.38%	263.362	1200
		12	520	349.934	48.60%	191.556	1200	409	344.478	18.73%	307.3	1200
		13	595	404.145	47.22%	196.215	1200	433	397.135	9.03%	356.988	1200
		14	608	406.331	49.63%	203.691	1200	464	398.362	16.48%	364.377	1200
		15	627	472.51	32.70%	190.102	1200	517	453.913	13.90%	292.425	1200
		16	560	418.268	33.89%	211.755	1200	441	401.471	9.85%	351.388	1200
		17	631	400.577	57.52%	222.372	1200	479	405.725	18.06%	338.889	1200
		18	658	421.375	56.16%	205.014	1200	474	405.996	16.75%	355.138	1200
		19	605	426.909	41.72%	223.721	1200	467	415.639	12.36%	391.566	1200
		20	646	425.802	51.71%	241.24	1200	517	420.383	22.98%	383.754	1200
		Average			**46.24%**					**14.85%**		
60	**150**	21	771	536.779	43.63%	416.218	1200	605	525.283	15.18%	1178.6	1200
		22	876	554.11	58.09%	452.763	1200	647	529.899	22.10%	1294.77	1200
		23	805	506.362	58.98%	442.192	1200	598	483.086	23.79%	1184.01	1200
		24	650	462.752	40.46%	383.721	1200	521	450.201	15.73%	960.061	1200
		25	594	425.674	39.54%	331.36	1200	475	419.451	13.24%	836.346	1200
		26	713	499.052	42.87%	391.344	1200	555	481.964	15.15%	1114.6	1200
		27	1034	637.276	62.25%	600.674	1200	802	599.379	33.81%	1603.16	1200
		28	684	458.603	49.15%	365.798	1200	513	444.661	15.37%	1085.64	1200
		29	661	487.255	35.66%	349.888	1200	532	478.645	11.15%	1008.83	1200
		30	612	432.078	41.64%	344.982	1200	486	428.267	13.48%	882.259	1200
		Average			**47.23%**					**17.90%**		
80	**200**	31	780	523.852	48.90%	589.704	1200	599	514.148	16.50%	1883.63	1200
		32	1133	668.192	69.56%	1592.79	1200	860	633.932	35.66%	4508.58	1200
		33	847	539.336	57.04%	836.084	1200	615	514.598	19.51%	2634.18	1200
		34	1153	688.514	67.46%	1069.28	1200	907	647.602	40.06%	2765.3	1200
		35	802	532.079	50.73%	710.028	1200	618	515.166	19.96%	2153.05	1200
		36	844	527.499	60.00%	724.343	1200	604	499.38	20.95%	2034.72	1200
		37	799	566.378	41.07%	636.31	1200	647	551.037	17.41%	1931.29	1200
		38	886	559.247	58.43%	754.596	1200	658	535.131	22.96%	2613.82	1200
		39	783	523.709	49.51%	616.943	1200	594	501.213	18.51%	1839.88	1200
		40	866	550.452	57.33%	808.613	1200	660	531.433	24.19%	2333.94	1200
		Average			**56.00%**					**23.57%**		

It was also desired to include a comparison between lower bounds which both are obtained through a Lagrangean relaxation and a subgradient optimization method, the only difference was the upper bound used at each iteration to calculate the step size, the CMI heuristic provided better upper bounds. The results of a linear relaxation are also included as this relaxation is also widely used to provide lower bounds. The results are interesting, despite of providing better upper bounds to calculate the step size the CMI was not able to improve the lower bounds. This was already mentioned but in tables 5 and 6 it becomes obvious that the differences are not very important and the lower bounds provided by the CMI are of almost the same quality. Also the lower bounds provided by the linear relaxation are way worse in every instance. This complies with the theory (Guignard, 2003) that says that Lagrangean relaxations provide equal or better lower bounds.

Table 5. Lower bounds of the Lagrangean Heuristic, the CMI Heuristic and linear Relaxation for Instances of class α and β

P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation	P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation
5	**50**	1	692.401	697.505	291.225	**12**	**50**	1	372.313	373.175	238.789
		2	739.914	739.931	310.8			2	407.985	407.709	256.985
		3	747.817	747.515	299.75			3	398.247	397.887	247.949
		4	649.846	648.95	303.574			4	364.83	363.883	237.493
		5	654.655	653.107	362.506			5	415.68	415.707	303.731
		6	777.999	772.088	369.443			6	464.371	463.043	311.568
		7	778.249	776.077	328.509			7	422.224	381.044	277.056
		8	769.808	769.369	299.962			8	382.879	389.119	257.711
		9	706.573	706.9	324.44			9	419.578	418.998	272.271
		10	802.97	801.8	318.324			10	430.328	431.758	273.751
10	**100**	11	998.272	985.311	411.707	**25**	**100**	11	526.792	513.719	340.005
		12	957.958	946.171	412.037			12	494.567	488.153	333.553
		13	1019.33	1017.06	475.226			13	533.682	532.021	393.776
		14	967.944	963.742	453.122			14	529.015	526.309	369.415
		15	1061.59	1063.08	469.944			15	567.449	562.183	388.482
		16	935.514	931.755	444.239			16	517.586	513.706	358.426
		17	1013.57	1011.06	427.145			17	533.946	527.646	353.368
		18	1028.4	1028.4	430.062			18	499.29	494.527	350.635
		19	1019.13	1000.21	464.085			19	530.451	529.983	383.502
		20	946.983	953.04	429.763			20	511.23	507.997	357.888

Table 5. Continuation

P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation	P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation
15	**150**	21	1275.16	1255.42	580.074	**37**	**150**	21	672.951	668.897	481.272
		22	1229.95	1198.78	530.573			22	639.438	624.721	444.526
		23	1265.47	1244.69	536.283			23	637.836	616.877	445.983
		24	1196.2	1181.57	507.416			24	583.936	580.621	418.215
		25	1182.66	1166.1	526.815			25	626.943	616.6	420.394
		26	1247.33	1237.1	595.161			26	643.201	635.593	489.976
		27	1300.1	1280.51	548.036			27	708.168	695.457	459.69
		28	1226.5	1192.31	514.709			28	632.093	627.037	419.275
		29	1215.27	1194.64	571.266			29	640.069	638.29	472.436
		30	1184.63	1172.17	502.353			30	618.012	613.298	406.685
20	**200**	31	1367.9	1345.19	620.612	**50**	**200**	31	709.348	696.301	504.557
		32	1393.61	1355.65	639.281			32	774.409	754.658	540.631
		33	1355.63	1321.63	599.483			33	691.898	682.254	494.342
		34	1364.77	1346.26	594.534			34	767.204	740.321	496.299
		35	1419.21	1390.39	594.636			35	723.974	706.577	488.432
		36	1371.66	1358.95	548.935			36	687.222	672.982	443.702
		37	1447.67	1418.97	658.808			37	743.203	739.859	539.862
		38	1365.24	1336.52	590.518			38	728.249	713.514	485.875
		39	1355.09	1337.3	620.286			39	709.347	705.536	499.116
		40	1408.88	1380.82	607.69			40	732.859	710.127	497.29

Table 6. Lower bounds of the Lagrangean Heuristic, the CMI Heuristic and linear Relaxation for Instances of class γ and Δ.

P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation	P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation
16	**50**	1	295.264	291.239	205.526	**20**	**50**	1	258.664	256.913	173.616
		2	327.992	324.7	227.967			2	291.755	270.348	198.41
		3	308.931	308.992	218.111			3	293.698	284.514	187.378
		4	296.067	290.503	201.72			4	261.358	253.674	173.842
		5	345.44	344.729	266.853			5	320.548	325.334	231.076
		6	381.416	381.376	276.038			6	357.775	342.065	243.089
		7	345.626	346.677	244.71			7	318.02	324.862	216.661
		8	311.581	312.999	228.141			8	282.596	271.253	204.65
		9	362.805	358.274	239.907			9	357.946	359.131	211.382
		10	365.668	365.886	243.5			10	345.721	350.993	220.026

Table 6. Continuation

P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation	P	N	Inst	Lagrangean LB	CMI LB	Linear Relaxation
33	**100**	11	405.29	396.55	296.28	**40**	**100**	11	392.2	386.834	264.697
		12	372.056	370.955	291.007			12	349.934	344.478	256.078
		13	437.458	435.276	345.034			13	404.145	397.135	306.129
		14	432.644	422.702	321.926			14	406.331	398.362	285.364
		15	467.248	459.346	340.395			15	472.51	453.913	303.978
		16	430.079	429.621	312.785			16	418.268	401.471	277.927
		17	422.883	418.067	310.345			17	400.577	405.725	276.77
		18	423.894	421.046	303.847			18	421.375	405.996	269.851
		19	428.617	423.924	335.754			19	426.909	415.639	300.057
		20	439.212	435.729	315.999			20	425.802	420.383	283.073
50	**150**	21	577.772	571.517	427.175	**60**	**150**	21	536.779	525.283	379.421
		22	533.571	521.715	395.532			22	554.11	529.899	352.458
		23	543.513	525.189	393.749			23	506.362	483.086	349.317
		24	494.854	487.53	366.678			24	462.752	450.201	323.222
		25	484.467	479.396	361.582			25	425.674	419.451	315.474
		26	524.938	522.74	427.987			26	499.052	481.964	380.043
		27	562.025	547.795	411.37			27	637.276	599.379	367.05
		28	498.73	489.759	365.38			28	458.603	444.661	320.742
		29	528.121	522.359	415.895			29	487.255	478.645	371.732
		30	486.616	480.523	350.982			30	432.078	428.267	306.592
66	**200**	31	569.572	561.916	437.62	**80**	**200**	31	523.852	514.148	385.314
		32	676.471	648.7	479.762			32	668.192	633.932	433.788
		33	595.084	579.776	432.245			33	539.336	514.598	383.067
		34	657.987	636.191	439.7			34	688.514	647.602	395.847
		35	589.528	578.133	427.817			35	532.079	515.166	381.074
		36	564.659	549.238	384.917			36	527.499	499.38	341.872
		37	615.46	612.299	472.403			37	566.378	551.037	419.706
		38	591.989	573.475	425.724			38	559.247	535.131	379.83
		39	572.354	565.873	431.734			39	523.709	501.213	377.609
		40	603.277	588.925	433.472			40	550.452	531.433	383.04