Chapter 1

Introduction

Nonlinear programming, sometimes known as nonlinear optimization, is the problem of maximizing or minimizing a function over a solution set of equalities and inequalities, collectively termed constraints, where some of these constraints or the objective function are nonlinear. Succinctly, nonlinear programming is the optimization of an outcome based on some set of constraints using a nonlinear mathematical model.

The purpose of this research is to broaden the nonlinear optimization theory by extending the results of D. Li and X. L. Sun's investigation found in [9]. Their results will be generalized from finite dimensional spaces to Hilbert spaces. We will show how to convexify locally the Lagrangian function of a transformed equivalent optimization problem. More precisely, we will prove that, under certain regular and second-order sufficiency conditions, the second Fréchet derivative of the p-power Lagrangian will become positive definite at an optimal point of the original problem for a sufficiently large p. Once we have done this, we will be able to use the duality theory to considerably broaden the scope of non-convex problems that can be solved through dual search methods.

This document will be organized as follows: in Chapter 2, we will review some important concepts for the development of our research. In Chapter 3, we will present the first and second-order necessary and sufficient optimality conditions, as well as the regularity conditions, given by Maurer and Zowe in [13] for infinite-dimensional programming problems with

constraints defined by arbitrary closed convex cones. In Chapter 4, we will present the results of D. Li and X. L. Sun regarding local convexification on finite dimensional spaces found in [9]. In this chapter, we will also expose the local convexity of the p-power Lagrangian. In Chapter 5, we will present the most important theorem of our investigation (theorem 5.1), which achieves our purpose of generalizing the results shown in the previous chapter to Hilbert spaces. In Chapter 6, we will present two numerical examples designed to illustrate that the p-power formulation indeed locally convexifies the Lagrangian function of the equivalent problem. Finally, conclusions will be given in Chapter 7.