Universidad de las Americas Puebla

School of Sciences

Actuary, Mathematics and Physics Department

Next-to-Leading-Order Accuracy Determination for Higgs Production in the Forward Region

Research dissertation presented by the student

Constanza Madero Antopia

164835

Overseen by Dr. Martin Hentschinski

San Andrés Cholula, Puebla.

Spring 2023

Contents

Abstract

The high-energy effective action is a theoretical framework which is used for the accurate description of Quantum Chromodynamics scattering amplitudes in the high energy limit. In particular it provides a field theory framework, which helps to systematically factorize QCD scattering amplitudes in the limit of high center of mass energies. In this project, we focus on the plus description for this effective action, and its physically significant results.

Keywords: Proton, KS Gluon, Forward Region, Plus Description.

Objective

The objective of this dissertation is to provide further context into how accurate calculations can work within the high energy factorization framework for further research in Quantum Chromodynamics.

1 Introduction

The world as we know it today is the product of the brilliant scientists that came before us. It's often hard to remember that not even 200 years ago, humanity had no idea what an electron even was – it wasn't until 1897 that Sir Joseph John Thomson proved their existence. Thomson's award-winning research consisted on a series of experiments on "the conduction of electricity by gasses" [2], where he demonstrated that cathode rays could be deflected by magnetic or electric fields; this was Thomson's cementing hint that cathode rays consist of charged particles. Then, by measuring the extent of the deflection of the cathode rays in electric or magnetic fields of various strengths, Thomson was able to calculate the mass-to-charge ratio of the particles [?]. These particles were emitted by the negatively charged cathode and repelled by the negative terminal of an electric field. Because like charges repel each other and opposite charges attract, Thomson concluded that the particles had a net negative charge; these particles are now called electrons. Most relevant to the field of chemistry, Thomson found that the mass-to-charge ratio of cathode rays is independent of the nature of the metal electrodes or the gas, which suggested that electrons were fundamental components of all atoms.

Joseph John Thomson was awarded the 1906 Nobel Prize in Physics *"in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases."* [2]. His discovery was the first time scientists forayed into the subatomic realm; afterward, in 1911 Ernest Rutherford identified discovered that atoms have a very dense nucleus - and, in 1919, he ascertained that these nuclei not only include electrons, but their positively-charged counterpart, the proton.

Several other conclusions were obtained thereafter, most relevant of which states that, if a nucleus is made up of particles of negative and positive charges, then they must repel each other - however, considering the infinitesimal (at the time) size of an atomic radius, this repulsion between particles must also be small enough for scale. This riddle was solved via the proposal of a brand new interaction, now called the *strong interaction*, or strong nuclear force, tasked with the burden of holding the universe together. And hold the universe together it does: the strong force is approximately 10^{38} times stronger than gravitation and about 100 times stronger than electromagnetism [23].

Nowadays, the fundamental theory of the strong nuclear force - also known, in context, as the *colour* force - is defined by interactions in terms of gluons and quarks, neatly packaged into the field known as Quantum Chromodynamics.

Quantum Chromodynamics is dedicated to describing the strong interactions of subatomic particles, characteristically that of gluon interactions between quarks. These interactions occur within the gluon field, a four-vector field which outlines the propagation of the previously mentioned gluon-quark interactions. Naturally, these synergies create a manner of binding energy, which, thanks to Einstein's infamous $E = mc^2$ energy-mass relation, means that composite particles called hadrons such as protons and neutrons - the 1932 [5] discovery of which was deeply impactful in the development of early 20th-century atomic physics - is directly generated by the strong interaction energy stored within the gluon field itself.

On an even smaller, $fundamental$ scale, quarks and electrons have also been been measured and demonstrated to have mass - which cultivates an additional field. Mathematically, bosons are required to be described as massless particles with a long-ranged force description. However, this requirement is empirically hindered, since weak and strong interactions are pointedly short-ranged and W and Z bosons have been known to be, in fact, massive. This is where the new field comes in: in the Standard Model of particle physics, a property mass generation mechanism for bosons exists, called the Higgs mechanism. [11]

The Higgs mechanism introduces a scalar quantum field, called the Higgs field -which, when subjected to quantum excitation, produces a Higgs boson. The existence of the Higgs boson (and therefore the Higgs field) had been solely theoretical until 2012, when experiments by the ATLAS and CMS at the Large Hadron Collider (LHC) observed the birth and death of a real-life Higgs boson in a groundbreaking, Nobel-prize winning discovery. [6]

The Higgs boson is a relatively novel elementary particle, produced by the quantum excitation of the Higgs field [19]; this boson is massive, scalar, and has null spin, electric charge, or colour charge. This is the latest discovered elementary particle, confirmed in 2012, and it's also the first observed elementary scalar particle [22].

The core investigative concept behind this thesis is to further study the elusive behaviour of the Higgs boson. For this, it's critical to have a precise understanding of not only its total production cross-section - that is, everywhere it can be generated after a collision, but also its distribution in transverse momentum and rapidity. Since the Higgs boson is produced through the impact of two gluons in the context of a proton-proton collision, it is apparent that the transverse momentum and rapdity distribution of the Higgs boson will strongly depend on the distribution of gluons in the proton.

To study this, Dr. Martin Hentschinski and associates considered [14] the Higgs production cross-section differential in rapidity and transverse momentum, which has been obtained within high energy factorization. The resulting expression uses a novel sort of plus description which provides a new mechanism to evaluate numerically certain convolution integrals; the goal of this thesis is to explore this proposed mechanism, and first arrive at a numerical implementation, and then verify whether it indeed provides numerically stable results.

The numerical study is based on the next-to-leading order expression derived within Dr. Martin Hentschinski et al's paper. In particular, we intend to make an implementation of the partonic coefficient of the equations derived in Mathematica, and to create a combination with parton distribution functions and unintegrated gluon distribution in the study of the transverse momentum dependence and of the numerical stability.

The novel observation of the Higgs boson by the ATLAS and CMS experiments confirmed have helped confirm that the Standard Model of particle physics is a consistent theory of strong and electro-weak interactions. The electro-weak sector of the Standard Model is now being explored in detail through the measuring properties of the Higgs boson.

The success of its discovery is complemented with an advancement in techniques for the calculation of production cross sections and decay rates with high accuracy; to determine the cross section for Higgs production in the dominant production central rapidity region, it's customary to use the framework for collinear factorization, where the incoming partons are collinear with the beam axis and are approximately on shell.

On the other hand, the description of more differential distributions give access to the region of small transverse Higgs momentum and extreme rapidities. The description of such processes requires the use of gluon distributions with dependence on the transverse momentum. From a phenomenological point of view, for the center of mass (COM) energies accessible at the Large Hadron Collider, cross-sections for forward production of Higgs bosons are most likely too small to be observed. Our result is, therefore, of formal interest. It serves to further explore the proper definition of NLO coefficients within high energy factorization [14].

This project generally focuses on a specific moment in photon-photon collisions within a specific case study, which shall be explained below. This project's goal is to build a mathematical simulation of parts of the interaction using Wolfram Mathematica; to do this, it is vastly important to outline a solid theoretical background in the particle interactions context.

In particle physics, the standard model is the current most complete model used for describing elementary particles; it is derived from a quantum field theory whose Lagrangian is divided in two sections: electroweak theory, which describes electromagnetic and weak forces, and quantum chromodynamics (QCD), which describes strong interaction. Each of these components describes a gauge spin 1 boson field, which propagates the interaction between spin fermion fields, with leptons in the first case and quarks in the other. Quarks and leptons are clasified in different flavour quantum numbers based on their mass and charge: quarks have quantum numbers corresponding to up, down, strange, charm, bottom and top, and leptons to the electron, muon, and tau, with their corresponding neutrinos. [1].

1.1 General characteristics of the standard model and fundamental particles

The establishing and definition of concepts pertaining to this dissertation is now presented, in order to make its digestion easier. All of the following definitions are given under the general particle physics umbrella.

- Elementary particle: An elementary, or fundamental, particle is a subatomic particle with no substructure; that is to say, it's a particle that is not composed of other, smaller particles [3].
- Proton: A subatomic particle with a positive electric charge, that is, a +1e elementary charge. It is one half of what constitutes an atomic nucleus [10].
- Quark: A type of elementary particle, and, in turn, a fundamental building block of matter. They combine to form composite particles called hadrons, such as protons and neutrons [20].
- Gluon: An elementary particle that serves as the exchange particle for the strong force between quarks. Analogous to the exchange of photons in the electromagnetic force between two charged particles [17].
- Collision: As per the classic definition, the sudden, forceful coming together in direct contact of two bodies [9].
- Higgs boson: The elementary particle associated with the Higgs field, which gives mass to certain other fundamental particles [7].
- Forward (direction): After a collision, the direction in which a particle is moving relative to the system.

These concepts constitute the basics needed to read on and understand with relative ease what is being discussed forward; other concepts may be briefly defined within the following text.

The Standard Model of particle physics is the general theory tasked with describing three of the four known universal fundamental forces (the electromagnetic force, weak interactions, and strong interactions). It also serves to classify all known elementary particles, and to predict various properties of weak, neutral currents, and the W and Z bosons extremely accurately. It's a show of global, interdisciplinary collaboration in modern times.

This model was and is still being developed in stages, mostly throughout the late 20th century. The current formulation was finalized in the mid-1970s, with the experimental demonstration of the existence of quarks. The confirmation of the Higgs boson existence in 2012 was the latest addition to the Model [18].

The Standard Model is believed to be theoretically self-consistent, although it leaves some unexplained phenomena and is still progressing towards being a full, theoretical explanation for fundamental interactions [15]. In particular, it fails to incorporate the full general relativity theory of gravitation, or account for the accelerating expansion of the Universe as possibly described by dark energy – in fact, it does not contain any viable dark matter particle with all required properties deduced from observational cosmology. It also does not incorporate neutrino oscillations and their non-zero masses. [21]

The development of the Standard Model is a grand collaboration between theoretical and experimental physicists alike. In the theory side of things, the Standard Model exhibits a wide range of phenomena, including spontaneous symmetry breaking, anomalies and non-perturbative behaviour [16]. It is used as a basis for incorporating hypothetical particles, extra dimensions, and elaborate symmetries [8]. All this is with the intention of explaining experimental results that are not necessarily congruent with the Standard Model, as was mentioned previously.

1.2 Proton-Proton Collisions

Moving forward, let us look into the difference between a 'normal' collision and the collision we're working on, with the forward-region; these diagrams, shown in Figure 1 and 2, are a representation of the two cases we're considering in this project; first, in Figure 1, we see the center case, where the collision of photons p_1^+ and p_2^+ occurs with equal rapidities from both sides, and result in the production of a Higgs boson upwardly, as seen:

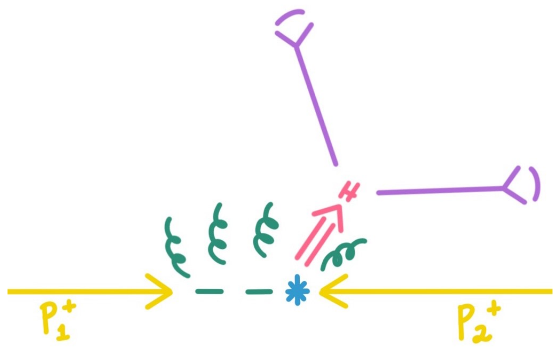

Figure 1: Higgs production in the center case.

What this means is that in the 'normal', or center, case, we have a faction x momentum, and in the forward direction (Figure 2) we have a dependence on the gluon's transverse momentum.

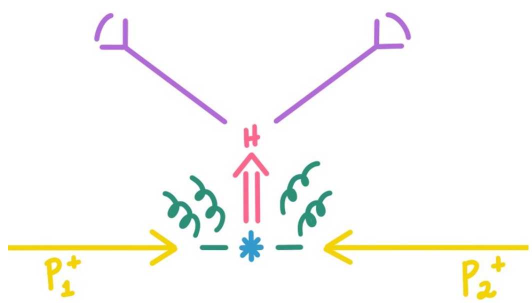

Figure 2: Higgs production in the forward case.

In the forward case, pictured in Figure 2, there's a very big difference in rapidity from each of the stages in the production of a Higgs boson, from the incoming photons, to the collision, to the scattering of the particles.

As stated previously, this thesis focuses on the study of a certain result which has been obtained within the framework of the so-called high energy effective action.

The high-energy effective action is a theoretical framework which is used for the accurate description of

Quantum Chromodynamics scattering amplitudes in the high energy limit. In particular it provides a field theory framework, which helps to systematically factorize QCD scattering amplitudes in the limit of high center of mass energies; see [4, 13] and references therein for a detailed discussion.

High-energy effective action is a term to define an attempt at a detailed description of scattering amplitudes, which exhibit regions where plus and minus momenta are strongly ordered. The high energy effective provides a field theory framework, which helps systematically factorize QCD (Quantum chromodynamic) scattering amplitudes and related theories in the limit of high center of mass energies.

1.3 The Plus Description within collinear factorization

Formally, the occurrence of a plus description is related to the theory of distributions, which in physics is more often talked about than actually studied. In a nutshell, distribution theory deals with objects (i.e., functions) which do not have well defined points, called singular points; there are also objects which have well defined integrals, while the distribution itself might have singular points.

An example is the delta function, $\delta(x)$, which is zero everywhere but infinity, when x=0; as a function, this is not very meaningful (for example, see, what does infinite mean?); it cannot really be used used at first, but then it becomes however a meaningful quantity,when we impose that the integral over the delta[x] is equal to one. In this same vein, distributions are functions which require integrals to make sense.

$$\int_{-\infty}^{\infty} dx\delta(x - x_0)f(x) = f(x_0) \tag{1}$$

Now, the plus description was first introduced for functions of the type $g(x) = f(x)/(1-x)$, with a finite f(1); however, when $g(1) \to \infty$ we have a problem: the following integral is not well defined.

$$\int_0^1 \frac{f(x)}{(1-x)} dx \tag{2}$$

It has the form $log(\infty) + c = \infty$, so it is, as previously states, undefined.

In Quantum Field Theory calculations, the following observation has been made: whenever an integral of the above type appears, there will also exist a corresponding contribution, which will yield a finite result whenever both terms are combined. Taking this into account, there exist two types of contribution:

- $$\frac{f(x)}{(1-x)} \tag{3}$$
- $$-\frac{f(1)}{(1-x)} \tag{4}$$

When both of these expressions are combined, we obtain:

$$h(x) = \frac{f(x) - f(1)}{(1-x)} \tag{5}$$

However, if we assume that $h(x)$ can be expanded as $h(x) = \frac{h(1)+h'(1)(x-1)+h''(1)}{2(1-x)^2+}$, we realise it is incorrect.

For $h(1)$ to be well defined, we must assume two things: One, that $f(x)$ can be expanded as $f(x) = f(1) + f'(1)(x-1) + f''(1)/2(1-x)^2+$, and two, that $h(1) = -f'(1)$.

This demonstrates that$h(x)$ is now a well defined distribution, over which we can integrate both analytically and numerically without any further complications. Thus, we can see that this so-called plus distribution is really just a description to define integral (2), hence the name "plus description".

From a technical point of view (that is, concerning actual numbers), the plus description is usually a good method to get a stable description; however, as an alternative, one could introduce some parameter ϵ and then integrate,

$$\int_0^{1-\epsilon} \frac{f(x)}{(1-x)} dx, - \int_0^{1-\epsilon} \frac{f(1)}{(1-x)} dx \tag{6}$$

In the end, both integrals would have had the $log(\epsilon)$ term, which we need to cancel out between them. Numerically, it tends to be hard to find a good value for ϵ; it must be a value that's small enough so the approximation is good, but big enough that the numerical part still work. In this case, the plus description is usually the superior method and gives stable results.

The specific plus description in this project does not exist in a traditional x space with x in (0,1); this plus description deals with transverse momentum integrals, which run from $(-\infty, \infty)$. So we need to modify the 'traditional' plus description. In this case, the more complicated version is used, since we also need to make sure that the prescription does not give us any brand new problems or unexpected troubles, especially within the Mathematica code, which is already fragile in itself.

2 Methodology

The goal of our plus description is to have mas much control as possible within a region $r \to 0$. We also need to make sure that our code and mathematical calculations are stable in the case when $r = p$, or, in its defect, $r = \infty$.

The goal of this thesis is, fundamentally, the numerical exploration of the plus description for transverse momentum space.

For this, we introduce the light-light momenta of the colliding protons p_1 and p_2 as $p_1^2 = p_2^2 = 0$; this is later used as a basis to express the relation of other momentas with respect to the momenta of these colliding protons. A generic momentum k can then be written as:

$$k = \alpha p_1 + \beta p_2 + k_\perp \tag{7}$$

Where $k_\perp = p_1 = 0$, p and $k_\perp$ each denote transverse momentum. If the collision axis is given by the z-axis, then $k_\perp^\mu = (0, k^x, k^y, 0)$, and we further propose that $k_\perp^2 - \vec{k}^2$, where $\vec{k}^2 = k^{x^2} + k^{y^2}$. This new definition of a momentum k was used to describe the plus description, with a $G(k)$ test function simulating Dr. Hentschinski's results of the unintegrated gluon distribution of high energy factorization.

This test function was best explored using Mathematica, and as such, the steps taken in the process of building and testing the Mathematica code will now be outlined; specifically, the code pertaining to the building blocks of what comes behind what we have previously defined as the plus description will be described, and then the plus description itself is to be arranged. The G(1) function resembles the shape of a typical unintegrated gluon distribution, and, since it is a decidedly *analytical* function, the integration should be easier than for an interpolated list.

Defining several important points was the first step taken; beginning with the theoretical and technical backgrounds in order to understand exactly what was going to be done over the course of the internship period. This process took most of the first semester, since it was a reduced period (this took place over the first summer semester of 2021, from May to June). The resulting theoretical framework is detailed above; please refer to the General characteristics section.

Next we set about establishing the mathematics. The plus description is part of a function of k^2, which comes from the Sudakov decomposition of a generic four momentum, wherein k_T is the embedding of the Euclidean vector k into Minkowski space, so $k_T^2 = k_2$. The function that's generated will subsequently be called G, which is a test function, and in application will be always the transverse momentum-dependent gluon distribution; we also came to define the variables $\delta = z\vec{r}(1z)\vec{p}$, with $\vec{r} = \vec{k} - \vec{p}$, which act solely as complimentary to the tests tasked.

The basic idea behind our plus description is to subtract a certain auxiliary term from the real NLO corrections, which both renders the latter finite and can be easily integrated analytically, and then added to the virtual NLO corrections [14]. The following is, then, the equation behind the project:

$$\int \frac{d^{2+2\epsilon}\vec{r}}{\pi^{1+\epsilon}} \frac{\kappa(\vec{r})}{\vec{r}^2} G((\vec{p}+\vec{r})^2) = \int \frac{d^2\vec{r}}{\pi} \left[\frac{\kappa(\vec{r})}{r^2}\right]_+ G((\vec{p}+\vec{r})^2) + \int \frac{d^{2+2\epsilon}\vec{r}}{\pi^{1+\epsilon}} \frac{\kappa(\vec{r})}{\vec{r}^2} \frac{\vec{p}^2 G(\vec{p}^2)}{\vec{r}^2 + (\vec{p}+\vec{r})^2} \tag{8}$$

3 Results

We of course began with the basics; the simplest part of this first plus description was developed through a Poisson model, which we defined by the function $G(k_2, \delta, Q\theta)$.

This model, after integration, gave us a list of values that became profitable in the process of arranging the way our Kutak-Sapota (from this point on referred to as ”KS”) Gluon would look like once implemented; this is basically the process of making a number of tables and plots in order to see how the plus description prediction would behave in a simpler context.

The generated Poisson model was ran three times with three distinct integrand values inputted; three different plots were generated, which demonstrated behaviour expected through Dr. Hentschinski’s previous research.

The following figures are examples of the Poisson model integration plots obtained:

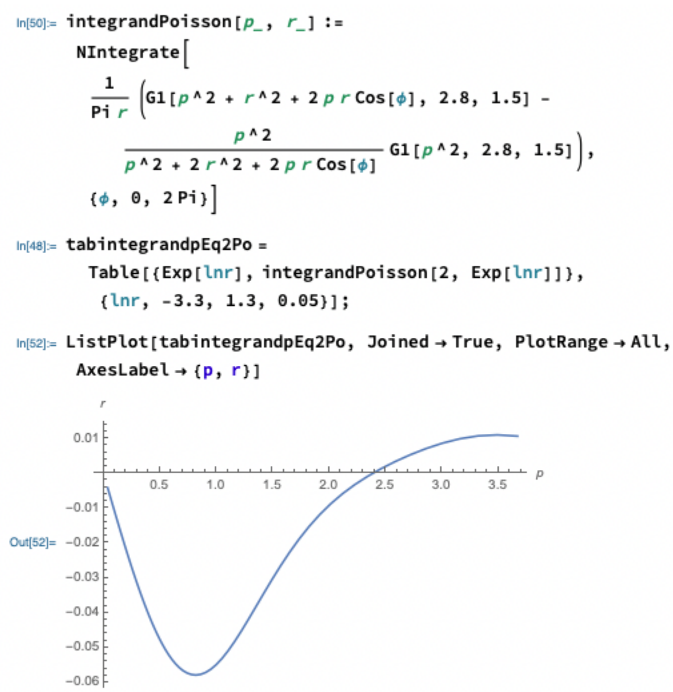

Figure 3: Mathematica code and plot of the Poisson integrand with standard values.

On this figure, figure 3, we consider the case when the function $\kappa(r) = 1$; from these results, we find that the resulting curve in the external momentum p is smooth throughout the entire range of relevant values of p. Therefore, we can conclude that this procedure works very well for these kinds of simpler studies.

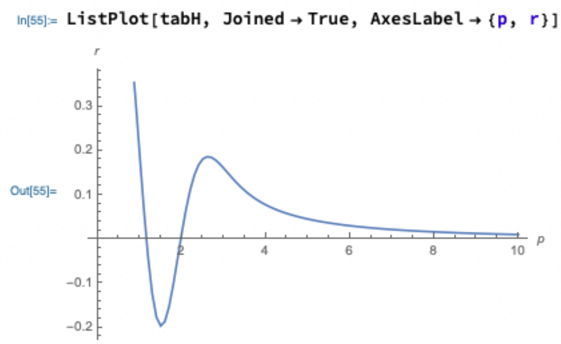

Figure 4: Plot of the Poisson integrand with values (p, 0.1, 10, 0.1)

For the above figure 4, where we introduce more specific values for the Poisson integrand, we turn to a more elaborate case, where $\kappa = \cos(\phi)$, wherein ϕ is the angle between p and r. This plot spikes, then dips, then spikes again around the 0.2r mark and tapers off smoothly at a more exponential rate throughout all relevant values of p. It's important to observe that these values are larger than those in the previous plot, which gives us a good, more accurate representation of what we're trying to achieve.

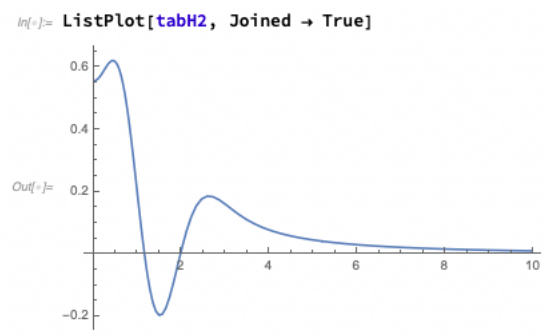

Figure 5: Plot of the Poisson integrand with values (log(p), -3.3, 2.3, 0.05)

Finally, on figure 5, we introduce our most complicated case yet: $\kappa = \log(r)$. Although its behaviour mimics the previous plot in a relatively faithful fashion, the logarithm for this integrand and its corresponding plot is divergent for $r \leftarrow 0$, which is feasible to lead into further instability. In this particular frame, though, the integrand and plot are satisfactory.

These plots and their corresponding tables tell us that the model works as it should, so it was safe to go forward.

3.1 Using the KS unintegrated gluon distribution

From this point, the project carried on testing with the full gluon distribution, for which it was necessary to first set up a KS and GBW model gluon (the latter of which was, again, used solely as a first tester before going into the plus-description proper). Both of these functions were compiled as transverse momentum-dependent gluons by one of Dr. Hentschinski's colleagues, Dr. Kutak. The GBW model gluon was used as an alternative gluon distribution, which was calculated from an unintegrated gluon distribution from the same model with values taken from the second table on a paper by Golec-Biernat and Sapeta [12].

As one could notice, up until now we'd been setting up and testing out different parts of the code in preparation for the main event; forward, the longest part as of yet was set in motion, which was testing out the three versions of the plus description for the KS Gluon. The three plus descriptions, with different kappa functions, used in Mathematica are as follows:

$$h_{KS}(p,x) = \int_{r,\phi}^{\infty,\pi} \frac{1}{2\pi r}\left(f(x, p^2 + r^2 + 2pr\cos\phi - \frac{p^2}{(p^2 + 2r^2 + 2pr\phi} f(x,p^2) \right) \tag{9}$$

This equation marks the $\kappa = 1$ case, which was expected and proven to be the simplest case to both map and describe.

$$\begin{aligned} h_{KS_2}(p,x) = \int_{r,\infty}^{\phi,2\pi} &\frac{\log(r^2)}{\pi r}\left(\frac{p^2(f(x,p^2 + r^2 + 2pr\cos\phi - f(x,p^2))}{(p^2 + 2r^2 + 2pr\phi} \right) \\ &+ \frac{\log(r^2)}{\pi r}\left(\frac{(2r + 2p\phi)(f(x,p^2 + r^2 + 2pr\cos\phi)}{(p^2 + 2r^2 + 2pr\phi} \right) \end{aligned} \tag{10}$$

This second equation is the Poisson case where $\kappa(r) = 1$; the behaviour of this plot was expected to be more volatile than the previous equation, with an increase in complexity thanks to the introduction of a logarithm and the extension required to make the $\kappa(r)$ case work.

$$\begin{aligned} h_{KS3}(p,x) = \int_{r,\infty}^{\phi,2\pi} &\frac{\cos(\phi^2)}{\pi r}\left(\frac{(f(x,p^2 + r^2 + 2pr\cos(\phi) - f(x,p^2)p^2)}{(p^2 + 2r^2 + 2pr(\phi)} \right) \\ &+ \frac{(\phi^2)}{\pi}\left(\frac{(2r + 2p\phi)(f(x,p^2 + r^2 + 2pr\cos\phi)}{(p^2 + 2r^2 + 2pr\phi} \right); \end{aligned} \tag{11}$$

$\kappa = \cos(\phi)$ defines our third and final, and most complicated, case, thanks to the combination of a logarithm and the introduction of an angled perspective. As mentioned previously, the logarithm in this case is divergent for $r \leftarrow 0$, which is feasible to lead into further instability.

From here, it was proceeded to have the most time-consuming part of the whole project; using the same methodology as previously described with the simpler Poisson integrands, the plus descriptions with the full gluon distribution was begun to be tested.

This was done using, as mentioned previously, basically the same idea behind the Poisson model but with the longer equations outlined above at (9), (10), and (11). A preliminary description was checked in a

Poisson KS integrand for (p, x, r), for which the table of values obtained was satisfactory and without much error; the following figure is a plot of the computed values.

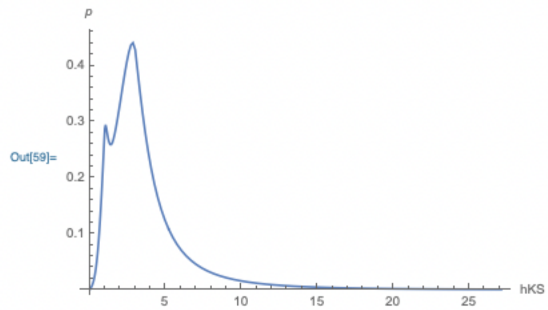

Figure 6: Plot of computed values of Poisson model for KS integrand and previously defined values for (-3.3, 3.3, .05).

Where $h_{KS}^{(1)}$ is our Poisson model KS Gluon distribution.

The tables of values obtained from the three defined plus descriptions were given distinct values to work over and outline, for which the Mathematica program became sluggish and difficult, since the equations are long and prone to error. These three equations outlined above were adjusted for precision at 3 and accuracy at 8, and defined in the regions of integraton $(0, p), (p, p + 2), \text{and}(p + 2, \infty)$.

The function $\kappa(\vec{r})$ from equation (8) is such that the integral on the right hand side of equation (8) is well-defined. It must also be such that the second integral in equation (8) can be calculated analytically [14]. There's three cases to be used for $\kappa(\vec{r})$, which are the ones that are being analysed in this project.

The first plus description, defined as $hKS1(p, x)$, is the 'standard' case for the description, with $\kappa(\vec{r}) = 1$ defined for equation (8). This description's tables behaved as expected and the Mathematica program gave no significant complaints. The following plot is the resulting culmination of values obtained and graphed.

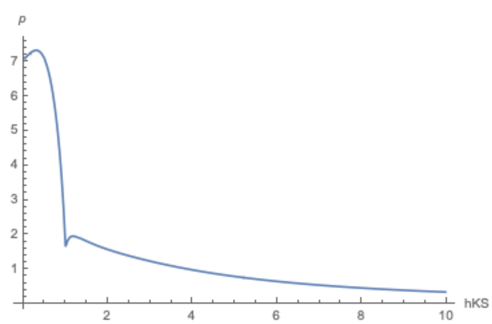

Figure 7: Plot of the values obtained for the first plus description model.

The second plus description, described as $hKS2(p, x)$, is the case wherein we have a function defined as $\kappa(\vec{r}) = \log(r^2)$ for equation (8). This table of values is where we began to have troubles, since it took a long time to compile and the Mathematica program spat out several error codes. The following plot is the resulting culmination of values obtained and graphed.

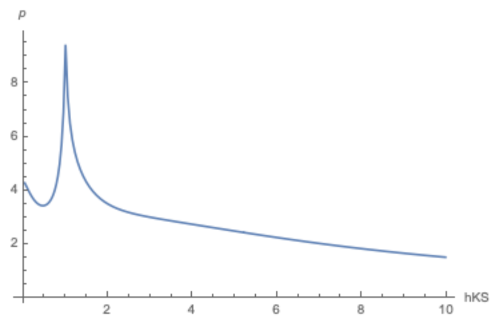

Figure 8: Plot of the values obtained for the second plus description model.

The third plus description, described as $hKS2(p, x)$, is the case wherein we have a function defined as $\kappa(\vec{r}) = (p \cdot r)^2/r^2)$ for equation (8). This table of values also resulted in the Mathematica program giving an array of error codes. The following plot is the resulting culmination of values obtained and graphed.

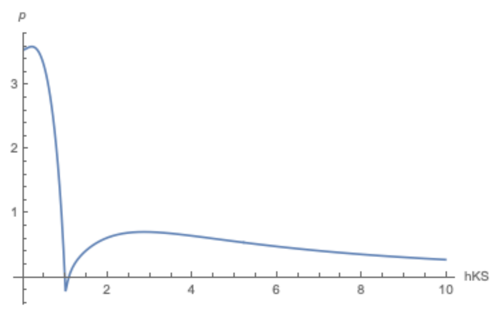

Figure 9: Plot of the values obtained for the third plus description model.

From the second and third plus description tables and plots, it was ascertained that our problems with the code begin if $p = 1$, which can be considered as a small value for a transverse momentum (p_T); we further tested the descriptions when $p > 1.5$, which is equivalent to $\log(p) > 0.4$.

The tables obtained for a large p_T were evaluated for values of the hKS_1, hKS_2 and hKS_3 at $(\log(p), 0.4, 4.3, 0.05)$. These values further showed that, in fact, the complications appear for the greater p_T region. It is theorised by Dr. Hentschinski that this is due to the integrand becoming too small.

In contrast to the previous results, it was observed that we obtained less complaints from the Mathematica program for the hKS_1 and hKS_3 than the second plus description, and these plots also looked very promising in terms of 'smoothness'. The following plots are the resulting culmination of values obtained and graphed.

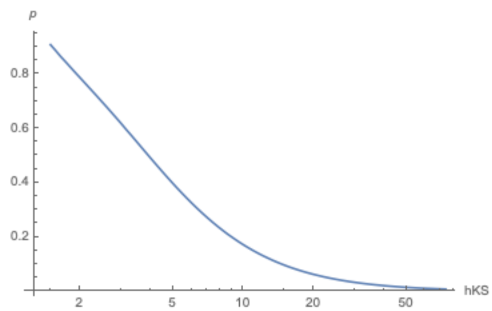

Figure 10: Plot of the values obtained for the first plus description model for the values $(\log(p), 0.4, 4.3, 0.05)$.

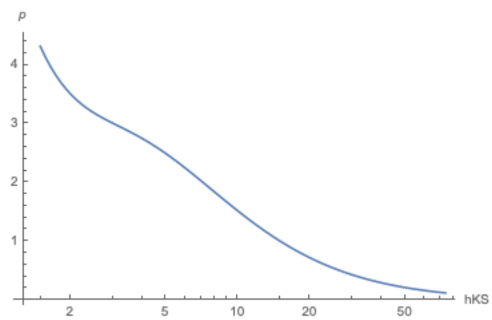

Figure 11: Plot of the values obtained for the second plus description model for the values $(\log(p), 0.4, 4.3, 0.05)$.

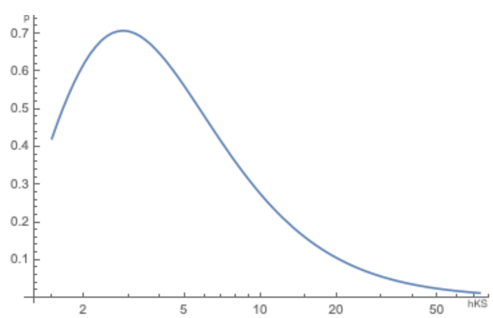

Figure 12: Plot of the values obtained for the third plus description model for the values $(\log(p), 0.4, 4.3, 0.05)$.

Finally, it was of interest to find a smoother line for the second plus description, so the function's Precision Goal was increased from the original 3 to 4. This precision goal was found to be the one that worked best, however, it was not entirely satisfactory. The following plots are the resulting culmination of values obtained and graphed.

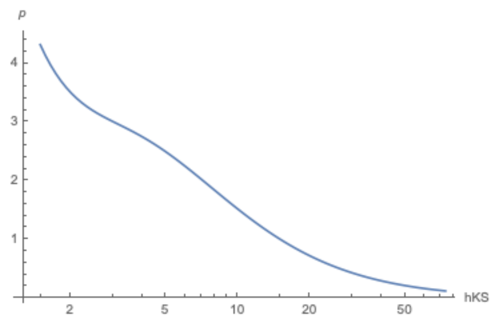

Figure 13: Plot of the values obtained for the second plus description model, adjusted with Precision Goal at 4.

The plus descriptions were further tested for errors; specifically, they were tested at $p_T max = e^{2.3}$, for which the Mathematica code gave no complaints, and $p_T max = e^{3.3}$, which gave some trouble but the results were stable.

4 Conclusions

In conclusion, within this project several important concepts in the field of quantum mechanics and particle physics were discussed. Quantum Chromodynamics were specifically outlined. The goal of this thesis was, fundamentally, to numerically explore a plus description case for transverse momentum space, and use the information obtained to deepen the analysis of using NLO accuracy to determine the production of a Higgs boson in the forward region post-collision. With this goal in mind, the light-light momenta of the colliding protons p_1 and p_2 as $p_1^2 = p_2^2 = 0$ were taken as a starting point; this is later used as a reference point from which to express other, more complex momenta with respect to the impetus of these colliding protons.

A proposed plus description term to make NLO corrections more functional was provided by Dr. Hentschinski and, over the course of the project, this defined plus description was worked on.

It is important to mention that for this whole project to have been possible, a plus description was absolutely necessary, as it is a superior method for finding values of ϵ when locked in a logarithmic function, especially when working from a technical point of view; it has been decidedly shown that a plus description layout consistently gives sturdy, stable results.

Formally, the occurrence of a plus description is related to the theory of distributions, which in physics is more often talked about than actually studied. In a nutshell, distribution theory deals with objects (i.e., functions) which do not have well defined points, called singular points; there are also objects which have well defined integrals, while the distribution itself might have singular points. In context, it was shown throughout this thesis that a certain plus distribution is really just a description to define a given integral, hence the name "plus description".

As mentioned previously, the specific plus description in this project deals with transverse momentum integrals, which run from $(-\infty, \infty)$. As such, the 'traditional' plus description was modified. In this case, a more complicated version was used, since it was also imperative to make sure that the description does not become unstable - the whole point of using a plus description was, of course, to make things simpler. The goal of our plus description was to have mas much control as possible within a region $r \to 0$. We also needed to make sure that our code and mathematical calculations were stable in the case when $r = p$, or, in its defect, $r = \infty$. This is the reason why the term $p^2/(r^2 + (r+p)^2)G(p)$ was added on.

Mathematica as a tool to study our mathematical findings was a choice well made, since it provides a trustworthy language in which to detail predictions without falling onto the redundant; it worked well in favour of the project.

The code built for this project was a collaboration, and over the course of the internship this project was worked on, several important results were obtained. These results showed that the provided plus description and its proposed working functions are workable within the Mathematica code, with a number of corrections attached. The plots resulting from these function evaluations are useful to the continuing work on the overall project, since they show the ways in which the plus descriptions will behave in an applied setting.

Looking into the future, the goal is to keep on studying distinct methodologies for increasing the efficiency and relative accuracy of our calculations; today, this task is well-performed by a plus description, but innovation is always on the horizon.

References

[1] PhD thesis.

[2] Nobel Prize Outreach AB. The nobel prize in physics 1906, 2023. https://www.nobelprize.org/prizes/physics/1906/summary/.

[3] S. Braibant, G. Giacomelli, and M. Spurio. *Particles and Fundamental Interactions: An Introduction to Particle Physics*. Springer, 2012.

[4] G. Chachamis, M. Hentschinski, J. D. Madrigal Martínez, and A. Sabio Vera. Forward jet production \& quantum corrections to the gluon Regge trajectory from Lipatov‘s high energy effective action. *Phys. Part. Nucl.*, 45(4):788–799, 2014.

[5] J. Chadwick. Existence of a Neutron. 06 1932.

[6] ATLAS Collaboration. Observation of a new particle in the search for the standard model higgs boson with the atlas detector at the lhc. 07 2012.

[7] CERN Collaborators. The higgs boson, 2021. URL: https://videos.cern.ch/record/2757407.

[8] CERN Collaborators. Standard model, 2021. URL: https://www.home.cern/science/physics/standard-model.

[9] Britannica Collaborators from the Encyclopedia Britannica. Collision, 2015. URL: https://www.britannica.com/science/collision.

[10] Britannica Collaborators from the Encyclopedia Britannica. Protons, 2019. URL: https://www.britannica.com/science/proton-subatomic-particle.

[11] M. Carena G. Bernardi and T. Junk. HIGGS BOSONS: THEORY AND SEARCHES . 11 2007.

[12] Krzysztof Golec-Biernata and Sebastian Sapeta. Saturation model of DIS: an update. 04 2018.

[13] Martin Hentschinski. Lipatov's QCD high energy effective action: past and future. 10 2020.

[14] Martin Hentschinski, Krzysztof Kutak, and Andreas Van Hameren. Forward higgs production within high energy factorization in the heavy quark limit at next-to-leading order accuracy. *The European Physical Journal C*, 81, 02 2021.

[15] Robert Mann. *An Introduction to Particle Physics and the Standard Model*, chapter 25. CRC Press, 1st edition, 2010.

[16] V. A. Miransky. *Dynamical Symmetry Breaking in Quantum Field Theories*, chapter 2. World Scientific Publising Co., 1st edition, 1993.

[17] C. R. Nave. Gluons, 2019. URL: http://hyperphysics.phy-astr.gsu.edu/hbase/Particles/expar.htmlc1.

[18] Robert Oerter and B. Holstein. The theory of almost everything: The standard model, the unsung triumph of modern physics. *Physics Today - PHYS TODAY*, 59, 07 2006.

[19] P. Onyisi. Higgs boson faq, 2012. University of Texas ATLAS group.

[20] James William Rohlf. *Modern Physics from a to Z0*. Wiley, 1994.

[21] PhD Sean Carroll. Caltech teaching company. In *Dark Matter, Dark Energy: The Dark Side of the Universe*, page 59, 2007.

[22] M. Strassler. The higgs faq 2.0, 2012. from ProfMattStrassler.com.

[23] M. Strassler. Particle physics basics, 2013. https://profmattstrassler.com/articles-and-posts/particle-physics-basics/the-known-forces-of-nature/the-strength-of-the-known-forces/.