Conclusions

As we have verified, the Lagrangian proposed in Chapter 3 equation (3.2) leads to the correct scalar field action in the CMPR framework. Furthermore, we have found a condition on the coupling constants, m_i, that predicts a family of actions that accomplish this task. We would like to point out the two special cases that one cannot avoid to notice.

The first case is that in which $m_2 = m_4 = 0$. This condition tells us that we can neglect the dual part of the action, and still manage to perform the scalar field coupling. In this case, the relation between m_1 and m_3 is given by $4!\alpha m_1^2 = m_3$. Let us choose, for simplicity $m_1 = 1$, then $m_3 = 4!\alpha$. Equation (3.2) would have the form

$$S_\varphi[B,\varphi,\pi] = \int\limits_{\mathcal{M}^4} \left[B_{IJ} \wedge B^{IJ}\pi^\mu\partial_\mu\varphi + 4!\alpha\varepsilon^{\alpha\beta\gamma\delta}B^{IJ}_{\mu\alpha}B_{\beta\gamma J}{}^{K}B_{\delta\nu KI}\pi^\mu\pi^\nu d^4x\right], \tag{3.22}$$

This expression is remarkably more simple than 3, and yet the equations of motion that result from it will be the same.

The second case is evidently that in which $m_1 = m_3 = 0$, meaning that we only want to keep the dual part of the action. The condition between m_2 and m_4 is given by $3![m_2(\alpha^2\sigma + \beta^2)]^2 = m_4\alpha^2\beta\sigma$. While this relation is more complicated than in the previous case, the purely dual action principle turns out to be relatively simple. For example, consider the particular case $m_2 = (\alpha^2\sigma + \beta^2)^{-1}$, then $m_4 = 3!\sigma/\alpha^2\beta$. This choice would lead to an action of the form

$$S_\varphi[B,\varphi,\pi] = \int\limits_{\mathcal{M}^4} \Big[\frac{1}{\alpha^2\sigma+\beta^2} B_{IJ}\wedge *B^{IJ}\pi^\mu\partial_\mu\varphi$$
$$+ \frac{3!\sigma}{\alpha^2\beta}\varepsilon^{\alpha\beta\gamma\delta}B^{IJ}_{\mu\alpha}B_{\beta\gamma J}{}^K * B_{\delta\nu KI}\pi^\mu\pi^\nu d^4x\Big]. \tag{3.23}$$

Another feature of the action that we have constructed, is that once one has established a good set of m_i, a rescalement of the coupling constants of the form $m_1 \to n\, m_1$ with $m_2 \to n\, m_2$, $m_3 \to n^2\, m_3$, and $m_4 \to n^2\, m_4$, where n is some scalar number, will also be a good set of coupling constants. This is by virtue of (3.20).

Finally, we would like to present a summary of the achievements of this work.

1. BF gravity was introduced in the first chapter of this work, starting out from the general structure of the theory: the BF term, and the restrictions. We have also pointed out that different restrictions give versatility to the theory and make it richer.

2. We looked at the form of the restriction of the CMPR action, which is a linear combination of two invariants. We argued that this restriction gives rise to a natural description of gravity with Immirzi parameter. On this matter, we studied the Holst action to give perspective on what the Immirzi parameter means, and pointed out the reason why BF gravity is important to quantum gravity.

3. The coupling of the cosmological constant was explained in detail, as first proposed by Montesinos and Velázquez, as this is a direct motivation to seeing how matter is coupled in the CMPR framework.

4. The coupling of the scalar field was studied in the context of general relativity. In order to do so, we looked at two different scalar field

actions that give rise to the same equations of motion. We showed this explicitly by computing the Klein-Gordon equation by performing the variation of the scalar field in the second action.

5. We analyzed a scalar field action in terms of the dynamical 2-forms of the BF type introduced in the CMPR framework. We partially showed that this action principle is equivalent to the one that is usually treated in the literature of general relativity. We also wrote down this action in terms of the tetrad field. As we said earlier, a detailed analysis of the coupling of matter fields (scalar, Maxwell, and Yang-Mills fields) to BF gravity including the Immirzi parameter is contained in [1]. This Bachelor thesis only contains a part of the results of [1] – those concerning the coupling of a scalar field – and has the goal of being an introduction to the issue of the couplings of matter fields in BF gravity.