Chapter 6

Method of Study of Ultra-peripheral collisions with decay of Di-Muons in ALICE experiment

In this chapter is shown the features and theoretical aspects of the ultra-peripheral collisions [34], [35], [36]. After it is explained the ultra-peripheral collisions, we explain the methology that is used to obtain data of ultra-peripheral collisions. Finally we include the decay of the dimuons in the methology of study.

6.1 Ultra-peripheral Collisions

The collisions that take place in the interaction point at ALICE can be separated in three categories. It depends on the impact parameter "b" that we can separete these collisions. The impact parameter is the distance between the center of the particles that will collide. It is shown in figure 6.1 the impact parameter "b".

- Central Collisions are the particular cases when the impact parameter is zero $b = 0$. This means that the collision is complete and not a fraction of the particles. In these cases the strong interaction is the one that takes place.

- Peripheral Collisions are the cases when a fraction of one of the particles collide with a fraction of the other. In order that this case takes place the collision parameter $R < b < 2R$.

- Ultra-peripheral Collisions are the cases of our interest. In these cases the particles do not collide, and the interaction is completly electromagnetic. However, the interaction is very strong between them, making the particles decay in others. The collission parameter is bigger than the sum of the radii of the particles $b > 2R$

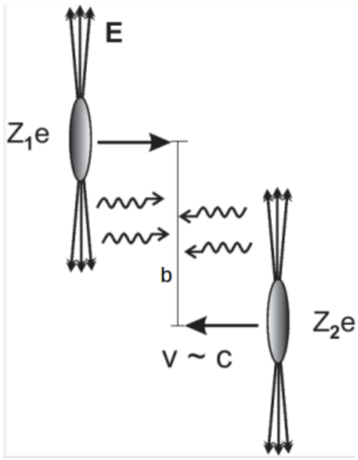

Figure 6.1: Ultraperipheral Collision: the interaction is completly electromagnetic. No hadronic interaction, since the ions pass nearby each other without traslaping. The impact parameter b is shown. It is the distance between the center of the ions.

The ultraperipheral collisions can lead to two different possibilities. The first one is that a photon is induced from one particle to the other via the electromagnetic interaction or both particles emit a photon and they collide producing a final product which we generalize calling *f*. The first possibility can be modelated by the *Weizäcker-Williams method*. In this method, the electric field lines are modelated as a *flux of virtual photons*. We give a general overview how these interactions of the virtual photons take place. Firstly, the cross section for photoproduction is as follows.

$$\sigma_X = \int d\omega \frac{n(\omega)}{\omega} \sigma_X^{\gamma}(\omega) \tag{6.1}$$

where $\sigma_X^{\gamma}(\omega)$ is the photonuclear cross section.

The equation above is for processes as the diagram in 6.2[1] a) i.e. a photon is induced to other particle and it is obtained a final product "X". The photon flux per unit area is

$$N(\omega, b) = \frac{Z^2 \alpha \omega^2}{\pi^2 \gamma^2 \hbar^2 (\beta c)^2} \left(K_1^2(x) + \frac{1}{\gamma^2} K_0^2(x) \right) \tag{6.2}$$

where $x = \frac{\omega b}{\gamma \hbar \beta c}$, the ion charge is Z, $\alpha = \frac{1}{137}$, βc is the velocity of the particle, i.e β is a fraction of the speed of light and K_0 and K_1 are modified Bessel functions. In order to obtain $n(\omega)$ in 6.1, we use

$$n(\omega) = \int N(w, b) db^2 \tag{6.3}$$

The equation above means "the sum of the differential distribution $\times$ differential area" give us the number of photons with certain energy. The term $K_1^2(x)$ is the

[1] Extracted from [34]

flux for longitudinally polarized photons. The transverse polarization dominates for ultra-relativistic particles $\gamma >> 1$. In photonuclear interaction (one photon) the usable photon flux is obtaned by integrating 6.2 over $b > b_{min}$ i.e. we force the flux to be for ultra-peripheral cases. We obtain

$$n(\omega) = \frac{2\xi Z^2 \alpha}{\pi \beta^2} \left[K_0(\xi) K_1(\xi) - \frac{\xi}{2} \left(K_1^2(\xi) - K_0^2(\xi) \right) \right] \tag{6.4}$$

where $\xi = \omega b_{min}/\gamma\beta\hbar c$. We express $\xi = 2\omega R_A/\gamma\beta\hbar c$, since $b_{min} = 2R_A$, i.e. the minimum impact parameter would be the sum of the two radii of the ions in order to be an ultraperipheral collision.

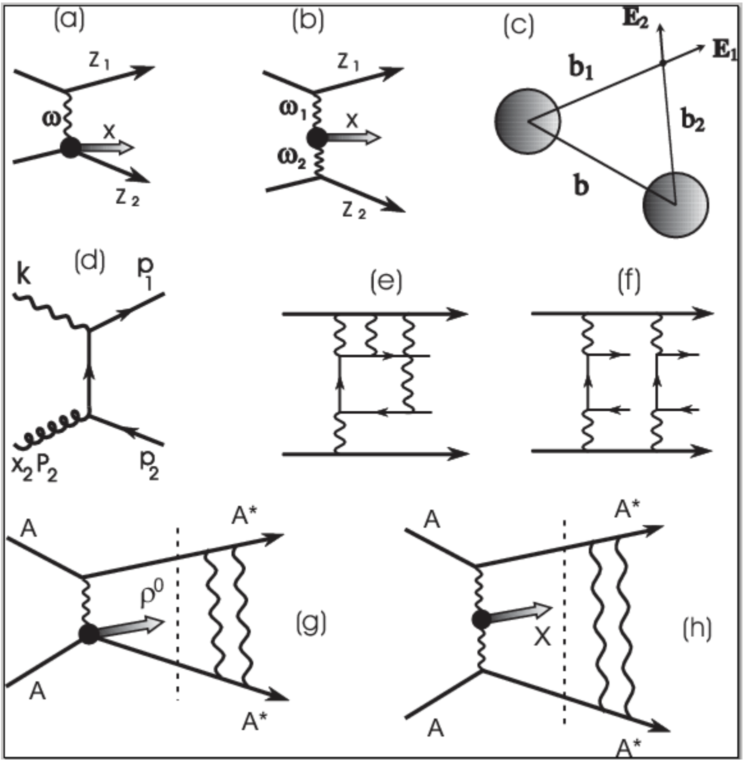

Figure 6.2: (a) One-photon and (b) two-photon processes in heavy ion collisions. (c) Geometrical representation of the photon fluxes at a point outside nuclei 1 and 2, in a collision with impact parameter b. The electric field of the photons at that point are also shown. (d) Feynman diagram for $q\overline{q}$ production through photon-gluon fusion to leading order. (e,f) Example of higher order corrections to pair-production: (e) Coulomb distortion, and (f) production of multiple pairs. (g) The dominant diagram for $Au + Au \to Au^* + Au^* + \rho^0$ and (h) for $Au + Au \to Au^* + Au^* + e^+e^-$ or a meson X. The dotted lines in panels (g) and (h) show how the mutual Coulomb nuclear excitation factorizes from the particle production.

For the particular case where two photons are emitted and they collide producing a new product "X' the cross section is given by the next equation.

$$\sigma_X = \int d\omega_1 d\omega_2 \frac{n(\omega_1) n(\omega_2)}{\omega_1 \omega_2} \sigma_X^{\gamma\gamma}(\omega_1 \omega_2) \tag{6.5}$$

where $\sigma_X^{\gamma\gamma}(\omega_1\omega_2)$ is the two photonuclear cross section. The equation above is for the cases shown in the Feynman Diagram 6.2 b). These interactions are principally mediated by pion exchange. The pion is a particle that can be either π^0,π^+ or π^-. These particles are the lightest mesons, where the mesons are particles composed of an anti-quark and quark particle. In order to obtain $n(\omega_1)$ and $n(\omega_2)$ we use equation 6.4. The cross-section obtained by this method has an error if the particle masses increases. More detailed calculations can be done by making integrals over b_1 and b_2 that could be weighted by hadronic non-interaction probability.

Equations 6.2 and 6.4 are valid only for electric dipole excitations. For the proton case, the proton size is included by the use of a form factor,since the measure is not exact. Also annexing a dipole form factor we obtian

$$n(\omega) = \frac{\alpha}{2\pi z}[1+(1-z)^2]\left(ln\chi - \frac{11}{6} + \frac{3}{\chi} - \frac{3}{2\chi^2} + \frac{1}{3\chi^3}\right) \tag{6.6}$$

where $\chi = 1 + \frac{0.71GeV^2}{Q^2_{min}c^2}$, $z = \frac{W^2}{s}$, W the γp center of mass energy, s the squared ion-ion center of mass energy per-nucleon and Q_{min} the minimum momentum transfer in the reaction. The equation above is valid when the proton remains intact. If the photon excitation is included the flux increases about 30%. If the photon energy is high i.e. when $z->1$ the magnetic form factor can become important

For the ion-ion ultraperipheral collisions the interaction time is $\Delta t \sim \frac{\gamma\hbar v}{b}$. And using Heisenberg uncertinty, in the laboratory frame the maximum photon energy is

$$\omega^{max} = \frac{\gamma\hbar v}{b} \tag{6.7}$$

where γ is the Lorentz factor $\gamma = (1-\frac{v^2}{c^2})^{-\frac{1}{2}}$. For grazling collisions, the impact parameter "b" is the sum of the radii of the nucleus(R_A). Sustituing b, we obtain that the maximum energy would be $\frac{\gamma\hbar v}{2R_A}$. The maximum photon energy is nearly $\frac{\hbar}{2R_A A_{m_p} c}$ of the ion energy, where A_{m_p} is the ion mass. The measure we have for protons radii is not precise, however it is reasonable to take as 10% of the proton energy as ω^{max}

In the ultraperipheral collisions, there exists two possible categories. The first one is called coherent interaction and the second one is called incoherent interaction. The first one mentioned is the case when the proton do not break, however by this interaction there is a new final product. The dominant coherent interaction leading to the production of a hadronic final state is the production of vector mesons.

$$\Lambda + \Lambda \longrightarrow \Lambda + \Lambda + V \tag{6.8}$$

As it was mentioned, the cross section can be calculated by equation 6.1. However it is generally impossible to know wich proton acted as targt and which was the photon emitter. In certain conditions they will interfere quantum mechanically. The cross section is given by adding the amplitudes A_1, A_2.

$$\frac{\hbar d\sigma}{dydp_T} = \int_{b>2R} |A_1 \pm |A_2|^2 d^2b \tag{6.9}$$

The Amplitudes A_1 and A_2 depend on the photon flux and on the photonuclear cross sections. Their p_T dependence comes from the convolution of the photon p_T spectrum and the p_T from the photon-nucleus scattering.

Since it is of our interest to obtain as the final product of the collision dimuons $\mu^+\mu-$, the cross section can be obtained similarly with the Dirac equation for the electron and its antiparticle

$$\sigma = \frac{28}{27\pi}\sigma_0[L^3 - 2.198L^2 + 3.821L - 1.632] \tag{6.10}$$

where $\sigma_0 = (Z_1Z_2\alpha^2\hbar/m_ec)^2$, $L = ln(\gamma_1\gamma_2)$, and γ_i is the Lorentz factor of ion i in the laboratory system. The replacement of $L \to \mathscr{L}$ =ln$(\gamma_1\gamma_2\delta/m_icR)$, where $\delta = 0.681...$ that is a number related to the Eulers constant and R is the nuclear radius,gives the cross section of the pair $\mu^+\mu-$.

The processes mentioned above were the particular case when there exists only one reaction per collision. However, it can exist the possibility that many reactions take place. Despite of the reactions might be independent, the geometry introduces correlations between the photon energies and polarizations. In multiphoton processes can be treated as independent if the photon emission does not excite the emitter. The cross section would be calculated as.

$$\sigma = \int d^2bP(b) \tag{6.11}$$

where $P(b)$ is the probability that the raction takes okace at certain impact parameter “b”.

$$P(b) = \int \frac{d\omega}{\omega}N(\omega, b)\sigma_{\gamma\Lambda}(\omega) \tag{6.12}$$

Ultraperipheral collisions are of interest, since new types of physics can be searched in these interactions. Some early calculations have focused on the search for the Higgs. Some other examples could be the supersymmetric particle pairs, magnetic monopoles and possible extra spatial dimensions. If supersymmetry is correct, a big amount of new particles could be present. Photonuclear interactions might be useful for studying supersymmetry.

6.2 Physical Variables of Particles to Measure

In Analysis of the collisions at CERN, we are intersted in some parameters or quantities that we can recover as data. As an example, suppose we have a collision, and by the detectors we are able to recover the tracks of the particles, product of the collision. By the track, we are able to know the angle of incidence of the particle in a plane. By this information we are able to calculate *pseudorapidity*, which can be used to obtain the momentum, and the momentum can be used to know the Energy. As this example, we can have some properties wich can be of our interest. In this subsection, we are intersted to introduce some important properties that are measured by the CERN's detectors, and how we use these different data to know about the information of the collisions.

6.2.1 Rapidity

Rapidity is a useful parameter to measure since it is additive under a longitudinal boost. We define rapidity as :

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} \iff y = \frac{1}{2}\ln\frac{c+v_x}{c-v_x} \tag{6.13}$$

Using the identity $\ln x = tan^{-1}(\frac{x^2-1}{x^2+1})$, we have:

$$y = tanh^{-1}\frac{(\sqrt{\frac{E+p_z}{E-p_z}})^2-1}{(\sqrt{\frac{E+p_z}{E-p_z}})^2+1} = tanh^{-1}\frac{\frac{E+p_z-(E-p_z)}{E-p_z}}{\frac{E+p_z+E-p_z}{E-p_z}} = tan^{-1}(\frac{p_z}{E}) \tag{6.14}$$

At the same time we have:

$$y = tanh^{-1}\frac{(\sqrt{\frac{c+v_z}{c-v_z}})^2-1}{(\sqrt{\frac{c+v_z}{c-v_z}})^2+1} = tanh^{-1}\frac{\frac{c+v_z-(c-v_z)}{c-v_z}}{\frac{c+v_z+c-v_z}{c-v_z}} = tan^{-1}(\frac{v_z}{c}) \tag{6.15}$$

As we have shown $\beta = \frac{v}{c}$(the constant that we use in Lorentz Transformations). So we can make $\beta = tanh(y)$. And making $cosh(y) = \gamma$, we obtain that $sinh(y) = \frac{v_z}{\sqrt{c^2+v^2}}$ Obtaining and recovering the boost transformations in direction z by the next linear transformation.

$$\begin{pmatrix} t' \\ z' \end{pmatrix} = \Lambda \begin{pmatrix} t \\ z \end{pmatrix} = \begin{pmatrix} cosh(y) & -sinh(y) \\ -sinh(y) & cosh(y) \end{pmatrix} \begin{pmatrix} t \\ z \end{pmatrix} \tag{6.16}$$

Where the determinant of the matrix Λ is 1 and an hyperbolic rotation.

As well, if we notice from the identity $\tanh(x+y) = \frac{\tanh(x)+\tanh(y)}{1+\tanh(y)\tanh(x)}$. We have that we can sum two pseudorapidities and it will behave similarly to the summation of the relativistic velocities,since $V = \frac{V_1+V_2}{1+\frac{V_1V_2}{c^2}}$

At the same time, rapidity is the analog of velocity in the non-relativistic limit ($p << m$),since:

$$y = \frac{1}{2}\ln\frac{\sqrt{p^2+m^2}+mv_z}{\sqrt{p^2+m^2}-mv_z} = \frac{1}{2}\ln\frac{m+mv_z}{m-mv_z} = \frac{1}{2}\ln(1+v_z) - \ln(1-v_z) \approx v_z$$

Where we used normalized speed of light as 1. Therefore, we found a quantity wich we call as *rapidity* wich can be used for boost rotations in a certain direction, wich follows the properties of the sum of relativistic velocities and a quantity wich is an analog of velocity in the non-relativistic limit.

6.2.2 Pseudo-Rapidity

If a particle path has a certain angle respect to a beam axis, which we stablish as our reference, we will have that moment $p_z = pcos\theta$ if we assume that the particle moves with a prefrential direction z. Therefore rapidity for a particle with a certain angle θ is:

$$y = \frac{1}{2}\ln\frac{E+pz}{E-pz} = \frac{1}{2}\ln\frac{\sqrt{p^2+m^2}+pcos\theta}{\sqrt{p^2+m^2}-pcos\theta} \tag{6.17}$$

Where we used the normalized speed of light as 1. If we consider that the particle is moving with a velocity close to speed of light,$i.e$ relativistic effect must be taken in account. The momentum p will be much greater than m. Therefore equation 6.17 becomes :

$$y = \frac{1}{2}\ln\frac{p+p\cos\theta}{p-p\cos\theta} = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = \frac{1}{2}\ln(\frac{1-\cos\theta}{1+\cos\theta})^{-1} = -\ln(\frac{1-\cos\theta}{1+\cos\theta})^{\frac{1}{2}}$$

And using the identity $\frac{1-\cos\theta}{1+\cos\theta} = \tan^2(\frac{\theta}{2})$, we obtain :

$$\eta = -\ln(\tan(\frac{\theta}{2})) \tag{6.18}$$

which we call as *pseudo-rapidity.*

Pseudorapidity is a powerful tool, since we can recover information about the kinematics of the particle with only the angle of it, and at the same time it is taking into account the relativistic effects, since we considered momentum p much bigger than the mass m. Pseudo-rapidity is the *rapidity* considering the case where the velocity is close to or considerebly close to speed of light.

6.2.3 Invariant Mass

From Special Relativity we know that the vectors are defined as cuadrivectors, where the first dimension is the time-dependent dimension and the other three are the spatial dimensions:

$$U^\mu = (u_t, u_x, u_y, u_z) = (u_0, u_1, u_2, u_3)$$

Therefore velocity vector must be as :

$$\mathbf{V} = (v_0, v_1, v_2, v_3) = (c, v_x, v_y, v_z)$$

At the same time the mass in Special Relativity is not just a scalar. It is affected, as space and time, by a factor $\gamma(v)$.

$$m = \gamma(v)m_0 = \frac{m_0}{(1-(v/c)^2)^{\frac{1}{2}}}$$

Therefore momentum is :

$$P = m\mathbf{v} = \gamma(v)m_0\mathbf{v} = \gamma(v)m_0\begin{bmatrix} c \\ v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} mc \\ mv_x \\ mv_y \\ mv_z \end{bmatrix}$$

And making $\mathbf{p} = (mv_x, mv_y, mv_z)$, we have that $P^\mu = (mc, \mathbf{p})$ If we take the product of P with itself, we obtain:

$$\begin{aligned} P^2 &= P^\mu \cdot P_\mu, \text{ where } P_\mu = (mc, -\mathbf{p}) \\ &= m^2c^2 - \mathbf{p}^2 \\ &= {m_0}^2\gamma(v)^2c^2 - (m_0\gamma(v)v)^2 \\ &= {m_0}^2\gamma(v)^2(c^2 - v^2) \\ &= {m_0}^2\frac{c^2}{c^2-v^2}(c^2 - v^2) \\ &= {m_0}^2c^2 \end{aligned}$$

Using that :

$$\gamma(v)^2 = \frac{1}{1-\frac{v^2}{c^2}} = \frac{1}{\frac{c^2-v^2}{c^2}} = \frac{c^2}{c^2-v^2}$$

As well we have :

$$\begin{aligned} m^2c^2 - \mathbf{p}^2 &= m_0^2c^2 \\ E^2 - \mathbf{p}^2c^2 &= {m_0}^2c^4 \\ m_0 &= \sqrt{\frac{E^2}{c^4} - \frac{\mathbf{p}^2}{c^2}} \end{aligned}$$

where we have used Einstein Equation that $E = mc^2$