Chapter 4

Decay $h^0 - > \tau \mu$

The goal of this chapter is to show the calculations of the possible branching ratio in the Minimal Supersymmetric Standard Model(MSSM) of the decay $h^0 - > \tau \mu$. We have already shown the Ansatz for the extension of the MSSM and respectively how the MSSM extends the Standard Model. We proceed in the first subchapter to calculate the Branching Ratio of $h^0 - > \tau \mu$ with this extension of the MSSM. We explain a generalized procedure of calculating the sixteen possibilities of decay with our Ansatz. In the second part of this chapter we show the plots of our calculation, making a random variation of all the free parameters of the branching ratio and make a comparison with the branching obtained with CMS paper. [10]

4.1 Decay $h^0 - > \tau \mu$ in MSSM extended in FV

The quantum correction will be done by one-loop with s-leptons. In figure 4.1 a generalization of the possible decays is shown. In these diagrams the particle that interacts with the τ , will be labeled with the momentum q_1 . Simultaneously the particle that interacts with the μ particle will be labeled with the momentum q_3 . The amplitudes of the different decays will have the labels j and k, where j is asigned to the particle that interacts with the μ particle. The labels j and k to the particle that interacts with the τ particle. The labels j and k take the integers 1, 2, 3, 4 and each number is related to a particle as follows.

$$\begin{array}{l}
1 \longrightarrow \tilde{\mu}_1 \\
2 \longrightarrow \tilde{\mu}_2 \\
3 \longrightarrow \tilde{\tau}_1 \\
4 \longrightarrow \tilde{\tau}_2
\end{array}$$
(4.1)

As an example, the Feynman diagram with the decay τ_2 with momentum q_1 and μ_1 with momentum q_3 , the amplitude will be represented by M_{14} .

The Branching Ratio will be given by the sum of the different contributions

of the possible Feynman diagrams, with one loop quantum correction.

$$\mathcal{BR}(h^0 - > \tau \mu) = \frac{\Gamma(h^0 - > \mu \tau)}{\Gamma_{tot}}$$
(4.2)

where

$$\Gamma(h^{0} - > \mu\tau) = \sum_{j,k} \left\{ \frac{1}{8\pi\hbar m_{h^{0}}} \int_{(m_{\tau} + m_{\mu})c^{2}} |M_{jk}|^{2} \frac{\delta(m_{h^{0}}c - \frac{E_{T}}{c})\rho}{E_{T}} dE_{T} \right\}$$
(4.3)

And

$$\rho = \frac{c\sqrt{m_{h^0}^4 + m_{\mu}^4 + m_{\tau}^4 - 2m_{h^0}^2 m_{\mu}^2 - 2m_{h^0}^2 m_{\tau}^2 - 2m_{\mu}^2}m_{\tau}^2}{2m_{h^0}}$$

$$(4.4)$$



Figure 4.1: Generalized Decay of $h^0 - > \tau \mu$, where $\mu_1, \mu_2, \tau_1, \tau_2$ are the s-leptons

We label as p to the momentum of the higgs boson, q_1,q_3 to one of the s-particles in the quantum correction loop and q_2 is the momentum of the Bino particle. We can express the momentums in terms of the others as follows.

$$p = k_2 + k_1 \text{ Conservation of momentum}$$

$$q_2 = q_3 - k_1$$

$$q_3 = p + q_1$$

$$(4.5)$$

In order to have all the expressions in one momentum of the loop, we isolate the expressions in terms of q_1, k_2 and k_1 . We know the momentums k_1, k_2 , since they are the momentums of the particles of the decay.

$$q_{2} = p + q_{1} - k_{1} \text{ We substitute } q_{3}$$

$$= k_{2} + k_{1} + q_{1} - k_{1} \text{ We substitute } p$$

$$= k_{2} + q_{1}$$

$$q_{3} = k_{2} + k_{1} + q_{1} \text{ We substitute } p$$

$$(4.6)$$

We make the amplitude of probability over all the possibilities of momentum. Therefore we make the integral of the amplitude over all the 4-dimensional space, since we do not know the value of q_1 . The amplitudes will be calculated by an integral over the momentums in the loop as follows.

$$M_{jk} = \int \frac{d^4 q_1}{(2\pi)^4} \times \bar{v}_\mu \times g_{\tilde{B}\tilde{f}_j\mu} \times P_{\tilde{B}}(q_2) \times g_{\tilde{B}\tilde{f}_k\tau} \times u_\tau(k_2) \times P_{\tilde{f}_k}(q_1) \times g_{h^0\tilde{f}_j\tilde{f}_k} \times P_{\tilde{f}_j}(q_3)$$

$$\tag{4.7}$$

, where $g_{\tilde{B}\tilde{f}_{j}\mu}$ represent the interaction of the s-letpton with the particle μ and $g_{\tilde{B}\tilde{f}_{k}\tau}$ the interaction of the s-lepton with the τ particle . $P_{\tilde{f}_{k}}(q_{1}), P_{\tilde{B}}(q_{2})$ and $P_{\tilde{f}_{j}}(q_{3})$ are the propagators. The term $g_{h^{0}\tilde{f}_{j}\tilde{f}_{k}}$ represents the higgs interaction with the s-leptons. In table 4.3 are shown the propagators that are of our interest, while the interactions are taken from Table 3.2 and Table 3.3.

As it is shown in Ec. 4.7, the integral will be done in terms of one of the internal momentums. We choose q_1 , however it could be realized by any of the internal momentums (q_1, q_2, q_3) . Therefore we use the momentum equations in 4.6 and substitute in the propagators. And we have consequently an expression as follows

$$M_{jk} = \int \frac{d^4 q_1}{(2\pi)^4} \times \bar{v}_{\mu}(k_1) \times g_{\tilde{B}\tilde{f}_{j}\mu} \times P_{\tilde{B}}(k_2 + q_1) \times g_{\tilde{B}\tilde{f}_k\tau} \times u_{\tau}(k_2) \times P_{\tilde{f}_k}(q_1) \times g_{h^0\tilde{f}_j\tilde{f}_k} \times P_{\tilde{f}_j}(k_2 + k_1 + q_1)$$

$$(4.8)$$

We will separate the integral in three expressions, in order to simplify all the calculations. The new expressions will be N_{jk} , D_{jk} and α_{jk} . The labels are given since N_{jk} will be a numerator, D_{jk} a denominator and α_{jk} couplings to the fraction $\frac{N_{jk}}{D_{jk}}$. We start with the following product, where we substitute the Progator of \tilde{B} taken from table 4.3 and we generalize the possible interactions from table 3.3.

$$\bar{v}_{\mu}g_{\tilde{B}\tilde{f}_{j}\mu}P_{\tilde{B}}(k_{2}+q_{1})g_{\tilde{B}\tilde{f}_{k}\tau}u_{\tau}(k_{2}) = \bar{v}_{\mu}(k_{1})\frac{ga_{1}}{4}tan\theta_{w}(n_{1}+n_{2}\gamma_{5})\frac{i(\not{k}_{2}+\not{q}_{1}+m_{\tilde{B}})}{(k_{2}+q_{1})^{2}-m_{\tilde{B}}^{2}}\frac{ga_{2}}{4}tan\theta_{w} \times (n_{3}+n_{4}\gamma_{5})u_{\tau}(k_{2}) \\
= \bar{v}_{\mu}(k_{1})\frac{ig^{2}a_{1}a_{2}}{16}tan^{2}\theta_{w}(n_{1}+n_{2}\gamma_{5})\frac{\not{k}_{2}+\not{q}_{1}+m_{\tilde{B}}}{(k_{2}+q_{1})^{2}-m_{\tilde{B}}^{2}} \times (n_{3}+n_{4}\gamma_{5})u_{\tau}(k_{2}) \\
= \bar{v}_{\mu}(k_{1})\frac{ig^{2}a_{1}a_{2}}{16}tan^{2}\theta_{w}\frac{1}{(k_{2}+q_{1})^{2}-m_{\tilde{B}}^{2}} \times N_{jk} \quad (4.9)$$

, where $n_1,\!n_2,\!n_3,\!n_4$ can take the integers 1,3 and

$$n_{1}(j) = \begin{cases} 1 & \text{if } j = 2, 3 \\ 3 & \text{if } j = 1, 4 \end{cases}$$

$$n_{2}(j) = \begin{cases} 1 & \text{if } j = 1, 4 \\ 3 & \text{if } j = 2, 3 \end{cases}$$

$$n_{3}(k) = \begin{cases} 1 & \text{if } k = 2, 3 \\ 3 & \text{if } k = 1, 4 \end{cases}$$

$$n_{4}(k) = \begin{cases} 1 & \text{if } k = 1, 4 \\ 3 & \text{if } k = 2, 3 \end{cases}$$

$$a_{1}(j) = \begin{cases} -c_{\varphi} & \text{if } j = 1, 2 \\ s_{\varphi} & \text{if } j = 3, 4 \end{cases}$$

$$a_{2}(k) = \begin{cases} -s_{\varphi} & \text{if } k = 1, 2 \\ -c_{\varphi} & \text{if } k = 3, 4 \end{cases}$$

$$(4.10)$$

$$N_{jk} = \bar{v}_{\mu}(k_{1})(n_{1} + n_{2}\gamma_{5})(k_{2} + \not{q}_{1} + m_{\tilde{B}} \times (n_{3} + n_{4}\gamma_{5})u_{\tau}(k_{2})$$

$$(4.11)$$

If we substitute Ec. 4.9 in Ec. 4.8, we obtain.

$$M_{jk} = \frac{g^2 a_1 a_2}{16} tan^2 \theta_w \int \frac{d^4 q_1}{(2\pi)^4} \times \frac{1}{(k_2 + q_1)^2 - m_{\tilde{B}}^2} \times N_{jk} \times P_{\tilde{f}_k}(q_1) \times g_{h^0 \tilde{f}_j \tilde{f}_k} \times P_{\tilde{f}_j}(k_2 + k_1 + q_1)$$

We now substitute the propagators $P_{\tilde{f}_k}(q_1)$, $P_{\tilde{f}_j}(k_2+k_1+q_1)$ written generalized.

$$M_{jk} = \frac{ig^2 a_1 a_2}{16} tan^2 \theta_w \int \frac{d^4 q_1}{(2\pi)^4} \times \frac{i}{(k_2 + q_1)^2 - m_{\tilde{B}}^2} \times N_{jk} \times \times \frac{i}{q_1^2 - m_{\tilde{f}_k}^2} \times g_{h^0 \tilde{f}_j \tilde{f}_k} \times \frac{i}{(k_2 + k_1 + q_1)^2 - m_{\tilde{f}_j}^2} = -\frac{ig_{h^0 \tilde{f}_j \tilde{f}_k} g^2 a_1 a_2}{16} tan^2 \theta_w \int \frac{d^4 q_1}{(2\pi)^4} \times \frac{1}{(k_2 + q_1)^2 - m_{\tilde{B}}^2} \times N_{jk} \times \times \frac{1}{q_1^2 - m_{\tilde{f}_k}^2} \times \frac{1}{(k_2 + k_1 + q_1)^2 - m_{\tilde{f}_j}^2}$$

$$(4.12)$$

If we label the following expressions as α_{jk} and D_{jk}

$$\alpha_{jk} = \frac{ig_{h^0\tilde{f}_j\tilde{f}_k}g^2 a_1 a_2}{16} tan^2 \theta_w \tag{4.13}$$

$$D_{jk} = [(k_2 + q_1)^2 - m_{\tilde{B}}^2][q_1^2 - m_{\tilde{f}_k}^2][(k_2 + k_1 + q_1)^2 - m_{\tilde{f}_j}^2] \quad (4.14)$$

We can express ${\cal M}_{jk}$ as

$$M_{jk} = \alpha_{jk} \int \frac{d^4 q_1}{(2\pi)^4} \frac{N_{jk}}{D_{jk}}$$
(4.15)

We expand the products in the expression of ${\cal N}_{jk}$

$$N_{jk} = \bar{v}_{\mu}(k_{1})(n_{1} + n_{2}\gamma_{5})(\not{k}_{2} + \not{q}_{1} + m_{\tilde{B}})(n_{3} + n_{4}\gamma_{5})u_{\tau}(k_{2})$$

$$= \bar{v}_{\mu}(k_{1})\{n_{1}(\not{k}_{2} + \not{q}_{1} + m_{\tilde{B}})n_{3} + n_{1}(\not{k}_{2} + \not{q}_{1} + m_{\tilde{B}})n_{4}\gamma_{5} + n_{2}\gamma_{5}(\not{k}_{2} + \not{q}_{1} + m_{\tilde{B}})n_{3}$$

$$+ n_{2}\gamma_{5}(\not{k}_{2} + \not{q}_{1} + m_{\tilde{B}})n_{4}\gamma_{5}\}u_{\tau}(k_{2})$$

$$(4.16)$$

We substitute

And we use the following properties of the Dirac matrixes

$$\gamma_5^2 = 1$$

$$\gamma_5 \gamma^\mu = -\gamma^\mu \gamma_5 \tag{4.17}$$

$$N_{jk} = \bar{v}_{\mu}(k_{1}) \Big\{ n_{1}(\gamma^{\mu}k_{2\mu} + \gamma^{\mu}q_{1\mu} + m_{\tilde{B}})n_{3} + n_{1}(\gamma^{\mu}k_{2\mu} + \gamma^{\mu}q_{1\mu} + m_{\tilde{B}})n_{4}\gamma_{5} + n_{2}\gamma_{5}(\gamma^{\mu}k_{2\mu} + \gamma^{\mu}q_{1\mu} + m_{\tilde{B}})n_{3} + n_{2}\gamma_{5}(\gamma^{\mu}k_{2\mu} + \gamma^{\mu}q_{1\mu} + m_{\tilde{B}})n_{4}\gamma_{5}\Big\} u_{\tau}(k_{2}) \\ = \bar{v}_{\mu}(k_{1}) \Big\{ n_{1}n_{3}(\gamma^{\mu}k_{2\mu} + \gamma^{\mu}q_{1\mu} + m_{\tilde{B}}) + n_{1}n_{4}\gamma_{5}(-\gamma^{\mu}k_{2\mu} - \gamma^{\mu}q_{1\mu} + m_{\tilde{B}}) + n_{2}n_{3}\gamma_{5}(\gamma^{\mu}k_{2\mu} + \gamma^{\mu}q_{1\mu} + m_{\tilde{B}}) \Big\} u_{\tau}(k_{2}) \\ = \bar{v}_{\mu}(k_{1}) \Big\{ n_{1}n_{3}(k_{2} + q_{1} + m_{\tilde{B}}) + n_{1}n_{4}\gamma_{5}(-k_{2} - q_{1} + m_{\tilde{B}}) + n_{2}n_{3}\gamma_{5}(k_{2} + q_{1} + m_{\tilde{B}}) \Big\} u_{\tau}(k_{2}) \\ = \bar{v}_{\mu}(k_{1}) \Big\{ (n_{1}n_{3} - n_{2}n_{4})(k_{2} + q_{1}) + (n_{1}n_{3} + n_{2}n_{4})m_{\tilde{B}} \\ + \gamma_{5} \{ (n_{1}n_{4} + n_{2}n_{3})m_{\tilde{B}} + (n_{2}n_{3} - n_{1}n_{4})(k_{2} + q_{1}) \} \Big\} u_{\tau}(k_{2})$$

$$(4.18)$$

We substitute N_{jk} in M_{jk}

$$M_{jk} = \alpha_{jk} \int \frac{d^4 q_1}{(2\pi)^4} \bar{v}_{\mu}(k_1) \bigg\{ (n_1 n_3 - n_2 n_4) (\not{k}_2 + \not{q}_1) + (n_1 n_3 + n_2 n_4) m_{\tilde{B}} \\ + \gamma_5 \{ (n_1 n_4 + n_2 n_3) m_{\tilde{B}} + (n_2 n_3 - n_1 n_4) (\not{k}_2 + \not{q}_1) \} \bigg\} u_{\tau}(k_2) \times \frac{1}{D_{jk}}$$

$$(4.19)$$

We take out from the integral the factors that does not depend of q_1

$$\begin{split} M_{jk} &= \alpha_{jk} \bar{v}_{\mu}(k_{1}) \Biggl\{ (n_{1}n_{3} - n_{2}n_{4}) (\not{k}_{2} \int \frac{d^{4}q_{1}}{(2\pi)^{4}D_{jk}} + \int \frac{d^{4}q_{1}\not{q}_{1}}{(2\pi)^{4}D_{jk}}) \\ &+ (n_{1}n_{3} + n_{2}n_{4}) m_{\tilde{B}} \int \frac{d^{4}q_{1}}{(2\pi)^{4}D_{jk}} + \gamma_{5} \Biggl\{ (n_{1}n_{4} + n_{2}n_{3})m_{\tilde{B}} \int \frac{d^{4}q_{1}}{(2\pi)^{4}D_{jk}} \\ &+ (n_{2}n_{3} - n_{1}n_{4}) (\not{k}_{2} \int \frac{d^{4}q_{1}}{(2\pi)^{4}D_{jk}} + \int \frac{d^{4}q_{1}\not{q}_{1}}{(2\pi)^{4}D_{jk}}) \Biggr\} \Biggr\} u_{\tau}(k_{2}) \\ &= \alpha_{jk} \bar{v}_{\mu}(k_{1}) \Biggl\{ (n_{1}n_{3} - n_{2}n_{4}) (\not{k}_{2}IC2 + IC1) + (n_{1}n_{3} + n_{2}n_{4})m_{\tilde{B}}IC2 \\ &+ \gamma_{5} \Biggl\{ (n_{1}n_{4} + n_{2}n_{3})m_{\tilde{B}}IC2 + (n_{2}n_{3} - n_{1}n_{4}) (\not{k}_{2}IC2 + IC1) \Biggr\} \Biggr\} u_{\tau}(k_{2}) \end{aligned}$$

$$(4.20)$$

where IC1 and IC2 are the two integral cases. IC1 is the integral with q_1 and IC2 is the integration of $\frac{1}{D_{jk}}$. We use the tool *FeynCalc* to calculate and evaluate these integrals. The result of integral IC1 is in Ec. 4.25 and IC2 is in Ec. 4.26 Now we use the Dirac Ec. which states

$$k_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$$

$$\bar{v}_\mu(k_1) k_1 = -\bar{v}_\mu(k_1) m_\mu$$
(4.21)

$$M_{jk} = \alpha_{jk}\bar{v}_{\mu}(k_1) \left\{ (n_1n_3 - n_2n_4)(m_{\tau}IC2 + IC1) + (n_1n_3 + n_2n_4)m_{\tilde{B}}IC2 + \gamma_5 \left\{ (n_1n_4 + n_2n_3)m_{\tilde{B}}IC2 + (n_2n_3 - n_1n_4)(m_{\tau}IC2 + IC1) \right\} \right\} u_{\tau}(k_2)$$

$$(4.22)$$

We call as S_{jk} to the scalar part of M_{jk} and P_{jk} to the pseudoscalar part.

$$S_{jk} = (n_1 n_3 - n_2 n_4)(m_\tau IC2 + IC1) + (n_1 n_3 + n_2 n_4)m_{\tilde{B}}IC2$$

$$P_{jk} = \gamma_5 \bigg\{ (n_1 n_4 + n_2 n_3)m_{\tilde{B}}IC2 + (n_2 n_3 - n_1 n_4)(m_\tau IC2 + IC1) \bigg\}$$

$$(4.23)$$

And therefore

$$M_{jk} = \alpha_{jk} \bar{v}_{\mu}(k_1) \{ S_{jk} + P_{jk} \} u_{\tau}(k_2)$$
(4.24)

We show in table 4.1 the sixteen scalar and pseudoscalar parts of the different M_{jk} . The following expressions are the two possible integrals in M_{jk} , where we

used FeynCalc for these results.

$$IC_{1} = \int d^{4}q_{1} \frac{q_{1}}{(2\pi)^{4}((q_{1}+k_{2})^{2}-m_{\tilde{B}}^{2})(q_{1}^{2}-m_{\tilde{f}_{k}}^{2})((q_{1}+k_{2}+k_{1})^{2}-m_{\tilde{f}_{j}}^{2}))} \\ = \frac{-i\pi^{2}}{[m_{h^{0}}^{2}-(m_{\mu}+m_{\tau})^{2}][m_{h^{0}}^{2}-(m_{\mu}-m_{\tau})^{2}]} \\ \times \left\{ -[k_{1}(m_{h^{0}}^{2}-m_{\mu}^{2}+m_{\tau}^{2})-k_{2}(m_{h^{0}}^{2}+m_{\mu}^{2}-m_{\tau}^{2})]B_{0hjk} \\ +[k_{1}(m_{h^{0}}^{2}-m_{\mu}^{2}-m_{\tau}^{2})-2m_{\mu}^{2}k_{2}]B_{0mbj} + [2m_{\tau}^{2}k_{1}+k_{2}(-m_{h^{0}}^{2}+m_{\mu}^{2}+m_{\tau}^{2})]B_{0tbk} \\ -\left\{ k_{1}[m_{\tilde{B}}^{2}(m_{h^{0}}^{2}-m_{\mu}^{2}+m_{\tau}^{2})+m_{h^{0}}^{2}(m_{\tau}^{2}-m_{\tau}^{2})+m_{\mu}^{2}m_{\tilde{f}_{k}}^{2}+m_{\mu}^{2}m_{\tau}^{2}-2m_{\tilde{f}_{j}}^{2}m_{\tau}^{2}+m_{\tilde{f}_{k}}^{2}m_{\tau}^{2}-m_{\tau}^{4}] \\ -k_{2}[m_{\tilde{B}}^{2}(m_{h^{0}}^{2}+m_{\mu}^{2}-m_{\tau}^{2})+m_{h^{0}}^{4}-m_{h^{0}}^{2}(m_{\mu}^{2}+m_{\tilde{f}_{j}}^{2}+2m_{\tau}^{2})+m_{\mu}^{2}m_{\tilde{f}_{k}}^{2}-2m_{\mu}^{2}m_{\tilde{f}_{k}}^{2}-m_{\mu}^{2}m_{\tau}^{2} \\ +m_{\tilde{f}_{j}}^{2}m_{\tau}^{2}+m_{\tau}^{4}]\right\}F_{c0}\right\}$$

$$(4.25)$$

$$IntegralCase2$$

$$IC_{2} = \int \frac{d^{4}q_{1}}{(2\pi)^{4}((q_{1}+k_{2})^{2}-m_{\tilde{B}}^{2})(q_{1}^{2}-m_{\tilde{f}_{k}}^{2})((q_{1}+k_{2}+k_{1})^{2}-m_{\tilde{f}_{j}}^{2}))} = i\pi^{2}F_{C0}$$

$$(4.26)$$

where we labeled as $m_{\tilde{f}_k}$ to the mass of the s-particle with the momentum q_1 and $m_{\tilde{f}_j}$ to the s-particle with momentum q_3 . Furthermore

$$B_{0tbk} = B_0(m_{\tau}^2, m_{\tilde{B}}^2, m_{\tilde{f}_k}^2)$$

$$B_{0hjk} = B_0(m_{h^0}^2, m_{\tilde{f}_j}^2, m_{\tilde{f}_k}^2)$$

$$B_{0mbj} = B_0(m_{\mu}^2, m_{\tilde{B}}^2, m_{\tilde{f}_j}^2)$$

$$F_{c0} = C0(m_{h^0}^2, m_{\mu}^2, m_{\tau}^2, m_{\tilde{f}_k}^2, m_{\tilde{f}_j}^2, m_{\tilde{B}}^2)$$
(4.27)

Expression of Intregal Case 1 4.25 has dependence of k_1 and k_2 . We eliminate this dependence using the dirac equations in 4.21 with the following operations. We start with equation 4.24

$$M_{jk} = \alpha_{jk} \bar{v}_{\mu}(k_1) \{ S_{jk} + P_{jk} \} u_{\tau}(k_2) = \alpha_{jk} \{ \bar{v}_{\mu}(k_1) S_{jk} u_{\tau}(k_2) + \bar{v}_{\mu}(k_1) P_{jk} u_{\tau}(k_2) \}$$
(4.28)

where

$$\bar{v}_{\mu}(k_{1})S_{jk}u_{\tau}(k_{2}) = \bar{v}_{\mu}(k_{1})\left\{ (n_{1}n_{3} - n_{2}n_{4})(m_{\tau}IC2 + IC1) + (n_{1}n_{3} + n_{2}n_{4})m_{\bar{B}}IC2 \right\} u_{\tau}(k_{2}) \\
= (n_{1}n_{3} - n_{2}n_{4})(m_{\tau}\bar{v}_{\mu}(k_{1})IC2u_{\tau}(k_{2}) + \bar{v}_{\mu}(k_{1})IC1u_{\tau}(k_{2})) \\
+ (n_{1}n_{3} + n_{2}n_{4})m_{\bar{B}}\bar{v}_{\mu}(k_{1})IC2u_{\tau}(k_{2})$$
(4.29)

We obtain in the equation above 4.29 the multiplication $\bar{v}_{\mu}(k_1)IC1u_{\tau}(k_2)$. We use the dirac equation 4.21 and remove the momentum k_1 and k_2 dependence of the Integral Case 1 4.25

$$\begin{split} \bar{v}_{\mu} & (k_{1})IC_{1}u_{\tau}(k_{2}) = \frac{-i\pi^{2}}{[m_{h^{0}}^{2} - (m_{\mu} + m_{\tau})^{2}][m_{h^{0}}^{2} - (m_{\mu} - m_{\tau})^{2}]} \\ \times & \bar{v}_{\mu}(k_{1}) \Bigg\{ - [k_{1}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) - k_{2}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2})]B_{0hjk} \\ + & [k_{1}(m_{h^{0}}^{2} - m_{\mu}^{2} - m_{\tau}^{2}) - 2m_{\mu}^{2}k_{2}]B_{0mbj} + [2m_{\tau}^{2}k_{1} + k_{2}(-m_{h^{0}}^{2} + m_{\mu}^{2} + m_{\tau}^{2})]B_{0tbk} \\ - & \Bigg\{ k_{1}[m_{\bar{B}}^{2}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) + m_{h^{0}}^{2}(m_{\tau}^{2} - m_{\bar{f}_{k}}^{2}) + m_{\mu}^{2}m_{\bar{f}_{k}}^{2} + m_{\mu}^{2}m_{\tau}^{2} - 2m_{\bar{f}_{j}}^{2}m_{\tau}^{2} + m_{\bar{f}_{k}}^{2}m_{\tau}^{2} - m_{\tau}^{4} \\ - & k_{2}[m_{\bar{B}}^{2}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2}) + m_{h^{0}}^{4} - m_{h^{0}}^{2}(m_{\mu}^{2} + m_{\bar{f}_{j}}^{2} + 2m_{\tau}^{2}) + m_{\mu}^{2}m_{\bar{f}_{k}}^{2} - 2m_{\mu}^{2}m_{\bar{f}_{k}}^{2} - m_{\mu}^{2}m_{\tau}^{2} \\ + m_{\bar{f}_{j}}^{2}m_{\tau}^{2} + m_{\tau}^{4}] \Bigg\} F_{c0} \Bigg\} u_{\tau}(k_{2}) \\ = & \frac{-i\pi^{2}}{[m_{h^{0}}^{2} - (m_{\mu} + m_{\tau})^{2}][m_{h^{0}}^{2} - (m_{\mu} - m_{\tau})^{2}]} \bar{v}_{\mu}(k_{1}) \Bigg\{ - [-m_{\mu}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) \\ - & m_{\tau}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2})]B_{0hjk} + [-m_{\mu}(m_{h^{0}}^{2} - m_{\mu}^{2} - m_{\tau}^{2}) - 2m_{\mu}^{2}m_{\tau}^{2}]B_{0mbj} \\ + & [-2m_{\tau}^{2}m_{\mu} + m_{\tau}(-m_{h^{0}}^{2} + m_{\mu}^{2} + m_{\tau}^{2})]B_{0tbk} - \Bigg\{ - m_{\mu}[m_{\bar{B}}^{2}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) \\ + & m_{h^{0}}^{2}(m_{\tau}^{2} - m_{\bar{f}_{k}}^{2}) + m_{\mu}^{2}m_{\bar{f}_{k}}^{2} + m_{\mu}^{2}m_{\tau}^{2} - 2m_{\bar{f}_{j}}^{2}m_{\tau}^{2} - m_{\tau}^{4}] - m_{\tau}[m_{\bar{B}}^{2}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2}) \\ + & m_{h^{0}}^{4}(m_{\tau}^{2} - m_{\bar{f}_{k}}^{2}) + m_{\mu}^{2}m_{\bar{f}_{k}}^{2} - 2m_{\mu}^{2}m_{\bar{f}_{k}}^{2} - m_{\mu}^{2}m_{\tau}^{2} + m_{\bar{f}_{j}}^{2}m_{\tau}^{2} + m_{\tau}^{4}] \Bigg\} F_{c0} \Bigg\} u_{\tau}(k_{2}) \\ (4.30) \end{aligned}$$

The expression above is general since we can introduce also the spinors in P_{jk} . From 4.28 we have $\bar{v}_{\mu}(k_1)P_{jk}u_{\tau}(k_2)$ which is expressed as

$$\bar{v}_{\mu}(k_{1})P_{jk}u_{\tau}(k_{2}) = \bar{v}_{\mu}(k_{1})\gamma_{5} \bigg\{ (n_{1}n_{4} + n_{2}n_{3})m_{\tilde{B}}IC2 + (n_{2}n_{3} - n_{1}n_{4})(m_{\tau}IC2 + IC1) \bigg\} u_{\tau}(k_{2}) \\
= (n_{1}n_{4} + n_{2}n_{3})m_{\tilde{B}}\bar{v}_{\mu}(k_{1})\gamma_{5}IC2u_{\tau}(k_{2}) + (n_{2}n_{3} - n_{1}n_{4})(m_{\tau}\bar{v}_{\mu}(k_{1})\gamma_{5}IC2u_{\tau}(k_{2}) \\
+ \bar{v}_{\mu}(k_{1})\gamma_{5}IC1u_{\tau}(k_{2}))$$
(4.31)

As it can be noticed that all terms of Integral Case 1 4.25 are multiplied by one of the slashed momentums (k_1, k_2) . Therefore all terms are multiplied by γ^{μ} since $k_1 = \gamma^{\mu} k_{1\mu}$ and $k_2 = \gamma^{\mu} k_{2\mu}$. Consequently $\gamma_5 IC1 = -IC1\gamma_5$ because from eq. 4.17 we know that $\gamma_5 \gamma^{\mu} = -\gamma^{\mu} \gamma_5$. We obtain

$$\bar{v}_{\mu}(k_1)P_{jk}u_{\tau}(k_2) = (n_1n_4 + n_2n_3)m_{\bar{B}}\bar{v}_{\mu}(k_1)\gamma_5 IC2u_{\tau}(k_2) + (n_2n_3 - n_1n_4)(m_{\tau}\bar{v}_{\mu}(k_1)\gamma_5 IC2u_{\tau}(k_2)) - \bar{v}_{\mu}(k_1)IC1\gamma_5u_{\tau}(k_2))$$

(4.32)

In the equation above we would introduce the spinor $v_{\mu}(k_1)$ in IC1 and eliminate the dependence of k_1 . Then we factorize once again the spinor $v_{\mu}(k_1)$ and rearrenge as we have the expression in eq. 4.32. Therefore the expression 4.30 is applied to the IC1 in S_{jk} and P_{jk} . Therefore the expression for IC1 is given by.

$$IC_{1} = \frac{-i\pi^{2}}{[m_{h^{0}}^{2} - (m_{\mu} + m_{\tau})^{2}][m_{h^{0}}^{2} - (m_{\mu} - m_{\tau})^{2}]} \left\{ - [-m_{\mu}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) - m_{\tau}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2})]B_{0hjk} + [-m_{\mu}(m_{h^{0}}^{2} - m_{\mu}^{2} - m_{\tau}^{2}) - 2m_{\mu}^{2}m_{\tau}]B_{0mbj} + [-2m_{\tau}^{2}m_{\mu} + m_{\tau}(-m_{h^{0}}^{2} + m_{\mu}^{2} + m_{\tau}^{2})]B_{0tbk} - \left\{ -m_{\mu}[m_{\tilde{B}}^{2}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) + m_{\mu}^{2}m_{\tilde{f}_{k}}^{2} + m_{\mu}^{2}m_{\tau}^{2} - 2m_{\tilde{f}_{j}}^{2}m_{\tau}^{2} + m_{\tilde{f}_{k}}^{2}m_{\tau}^{2} - m_{\tau}^{4}] - m_{\tau}[m_{\tilde{B}}^{2}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2}) + m_{h^{0}}^{4} - m_{h^{0}}^{2}(m_{\mu}^{2} + m_{\tilde{f}_{j}}^{2} + 2m_{\tau}^{2}) + m_{\mu}^{2}m_{\tilde{f}_{k}}^{2} - 2m_{\mu}^{2}m_{\tilde{f}_{k}}^{2} - m_{\mu}^{2}m_{\tau}^{2} + m_{\tilde{f}_{j}}^{2}m_{\tau}^{2} + m_{\tilde{f}_{j}}^{2}m_{\tau}^{2} + m_{\tau}^{4}]\right\}F_{c0}\right\}$$

$$(4.33)$$

We continue simplifying the expression of the Integral Case 1 in order to make more clear the calculations. We will make three new expressions labeled as \mathcal{B}_{jk} (expression with all the B0 functions), C_{jk} and $C_{h^0\mu\tau}$. Firstly

$$\mathcal{B}_{jk} = -[-m_{\mu}(m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2})
- m_{\tau}(m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2})]B_{0hjk} + [-m_{\mu}(m_{h^{0}}^{2} - m_{\mu}^{2} - m_{\tau}^{2}) - 2m_{\mu}^{2}m_{\tau}]B_{0mbj}
+ [-2m_{\tau}^{2}m_{\mu} + m_{\tau}(-m_{h^{0}}^{2} + m_{\mu}^{2} + m_{\tau}^{2})]B_{0tbk}$$
(4.34)

If we factorize the common products of the masses, we obtain

$$\mathcal{B}_{jk} = m_{h^0}^2 m_{\mu} [B_{0hjk} - B_{0mbj}] + m_{h^0}^2 m_{\tau} [B_{0hjk} - B_{0tbk}]
- m_{\mu}^3 [B_{0hjk} - B_{0mbj}] - m_{\tau}^3 [B_{0hjk} - B_{0tbk}] + m_{\mu} m_{\tau}^2 [B_{0hjk} + B_{0mbj} - 2B_{0tbk}]
+ m_{\tau} m_{\mu}^2 [B_{0hjk} - 2B_{0mbj} + B_{0tbk}]$$
(4.35)

The result of the expression above is extremely important. The B0 function has a divergent term Δ_{UV} which needs to be eliminated in order to obtain a finite result. It is important to have the same number of B0 functions in the calculation expression with postive sign and negative sign in order to eliminate all the divergences.¹ In equation 4.35 it is possible to verify that all divergences will be eliminated. It is an important and remarkable result since it means that our loop correction does not need more contributions in order to be finite. Similarly we label

$$C_{h^0\mu\tau} = [m_{h^0}^2 - (m_\mu + m_\tau)^2][m_{h^0}^2 - (m_\mu - m_\tau)^2]$$
(4.36)

And

$$C_{jk} = -m_{\mu} \left[m_{\tilde{B}}^{2} (m_{h^{0}}^{2} - m_{\mu}^{2} + m_{\tau}^{2}) + m_{h^{0}}^{2} (m_{\tau}^{2} - m_{\tilde{f}_{k}}^{2}) + m_{\mu}^{2} m_{\tilde{f}_{k}}^{2} \right. \\ + m_{\mu}^{2} m_{\tau}^{2} - 2m_{\tilde{f}_{j}}^{2} m_{\tau}^{2} + m_{\tilde{f}_{k}}^{2} m_{\tau}^{2} - m_{\tau}^{4} \right] \\ - m_{\tau} \left[m_{\tilde{B}}^{2} (m_{h^{0}}^{2} + m_{\mu}^{2} - m_{\tau}^{2}) + m_{h^{0}}^{4} - m_{h^{0}}^{2} (m_{\mu}^{2} + m_{\tilde{f}_{j}}^{2} + 2m_{\tau}^{2}) + m_{\mu}^{2} m_{\tilde{f}_{j}}^{2} - 2m_{\mu}^{2} m_{\tilde{f}_{k}}^{2} - m_{\mu}^{2} m_{\tau}^{2} \right. \\ + m_{\tilde{f}_{j}}^{2} m_{\tau}^{2} + m_{\tau}^{4} \right]$$

$$(4.37)$$

 1 The finite expression from any substraction pair of B0 functions is given in Appendix. B

Using Ec.4.35, 4.36 and 4.37 we can express the generalized and final result for 4.33 as

$$IC1 = -\frac{i\pi^2}{C_{h^0\mu\tau}} \{ \mathcal{B}_{jk} - F_{c0}C_{jk} \}$$
(4.38)

Now that we have the general expression for M_{jk} and its respective terms we proceed to calculate $|M_{jk}|^2$ that is given by

$$|M_{jk}|^2 = M_{jk} M_{jk}^{\dagger} \tag{4.39}$$

$$|M_{jk}|^{2} = M_{jk}M_{jk}^{T}$$

$$= \{\bar{v}_{\mu}(k_{1})\alpha_{jk}(S_{jk} + P_{jk})u_{\tau}(k_{2})\}\{\bar{v}_{\mu}(k_{1})\alpha_{jk}(S_{jk} + P_{jk})u_{\tau}(k_{2})\}^{\dagger}$$

$$= |\alpha_{jk}|^{2}\{\bar{v}_{\mu}(k_{1})(S_{jk} + P_{jk})u_{\tau}(k_{2})u_{\tau}^{\dagger}(k_{2})(S_{jk}^{\dagger} + P_{jk}^{\dagger})\bar{v}_{\mu}^{\dagger}(k_{1})\}$$

$$= |\alpha_{jk}|^{2}\{\bar{v}_{\mu}(k_{1})(S_{jk} + P_{jk})u_{\tau}(k_{2})\}u_{\tau}^{\dagger}(k_{2})(S_{jk}^{\dagger} + P_{jk}^{\dagger})\gamma^{0}v_{\mu}(k_{1})\}$$

$$(4.40)$$

 S_{jk} is composed of complex numbers multiplied by the unit matrix. Therefore we can commute γ^0 . Furthermore, P_{jk} has complex numbers and multiplied by the γ_5 matrix. The complex numbers are not affected by the commutation of γ^0 , however $\gamma_5\gamma^0 = -\gamma^0\gamma_5$. Consequently we obtain

$$|M_{jk}|^{2} = |\alpha_{jk}|^{2} \{ \bar{v}_{\mu}(k_{1}) (S_{jk} + P_{jk}) u_{\tau}(k_{2}) \bar{u}_{\tau}(k_{2}) (S_{jk}^{\dagger} - P_{jk}^{\dagger}) v_{\mu}(k_{1}) \}$$

$$(4.41)$$

Since it is not known the final states of the spins of the particles μ and τ , we make an average of the possible states. Proceeding firstly with the average of the final state of $\bar{u}_{\tau}(k_2)$ and $u_{\tau}(k_2)$

$$|M_{jk}|^{2} = |\alpha_{jk}|^{2} \{ \bar{v}_{\mu}(k_{1}) (S_{jk} + P_{jk}) \frac{1}{2} \sum_{spin=1,2} \{ u_{\tau}(k_{2}) * \bar{u}_{\tau}(k_{2}) \} (S_{jk}^{\dagger} - P_{jk}^{\dagger}) v_{\mu}(k_{1}) \}$$

$$= \frac{1}{2} |\alpha_{jk}|^{2} \{ \bar{v}_{\mu}(k_{1}) (S_{jk} + P_{jk}) [k_{2} + m_{\tau}] (S_{jk}^{\dagger} - P_{jk}^{\dagger}) v_{\mu}(k_{1}) \}$$

(4.42)

We label as $Q = (S_{jk} + P_{jk}) [k_2 + m_{\tau}] (S_{jk}^{\dagger} - P_{jk}^{\dagger})$, to simplify and clarify the calculations.

We proceed to sum over the spins of $\bar{v}_{\mu}(k_1)$ and $v_{\mu}(k_1)$. Also, the multiplication

40

of matrixes will be expressed as the sumation notation.

$$|M_{jk}|^{2} = \frac{1}{2} |\alpha_{jk}|^{2} \{ \bar{v}_{\mu}(k_{1})Q_{jk}v_{\mu}(k_{1}) \}$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} \sum_{spin=1,2}^{\infty} \sum_{i,j=1}^{4} \bar{v}_{\mu}(k_{1})_{i}Q_{i,j}v_{\mu}(k_{1})_{j}$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} \sum_{i,j=1}^{4} Q_{ij} \sum_{spin=1,2}^{\infty} \{ v_{\mu}(k_{1})\bar{v}_{\mu}(k_{1}) \}_{ji}$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} \sum_{i,j=1}^{4} Q_{ij} [k_{1} - m_{\mu}]_{ji}$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} \sum_{i=1}^{4} [Q(k_{1} - m_{\mu})]_{ii}$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} Tr[Q(k_{1} - m_{\mu})]$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} Tr[(S_{jk} + P_{jk})(k_{2} + m_{\tau})(S_{jk}^{\dagger} - P_{jk}^{\dagger})(k_{1} - m_{\mu})]$$

$$(4.43)$$

Expanding the products

$$|M_{jk}|^{2} = \frac{1}{4} |\alpha_{jk}|^{2} Tr [\{ (S_{jk} + P_{jk}) \not{k}_{2} S_{jk}^{\dagger} - (S_{jk} + P_{jk}) \not{k}_{2} P_{jk}^{\dagger} + (S_{jk} + P_{jk}) m_{\tau} S_{jk}^{\dagger} - (S_{jk} + P_{jk}) m_{\tau} P_{jk}^{\dagger} \} (\not{k}_{1} - m_{\mu})]$$

$$= \frac{1}{4} |\alpha_{jk}|^{2} Tr [(S_{jk} + P_{jk}) \not{k}_{2} S_{jk}^{\dagger} \not{k}_{1} - (S_{jk} + P_{jk}) \not{k}_{2} P_{jk}^{\dagger} \not{k}_{1} + (S_{jk} + P_{jk}) m_{\tau} S_{jk}^{\dagger} \not{k}_{1} - (S_{jk} + P_{jk}) m_{\tau} P_{jk}^{\dagger} \not{k}_{1} - (S_{jk} + P_{jk}) \not{k}_{2} S_{jk}^{\dagger} m_{\mu} + (S_{jk} + P_{jk}) \not{k}_{2} P_{jk}^{\dagger} m_{\mu} - (S_{jk} + P_{jk}) m_{\tau} S_{jk}^{\dagger} m_{\mu} + (S_{jk} + P_{jk}) m_{\tau} P_{jk}^{\dagger} m_{\mu}]$$

$$(4.44)$$

If we consider the following properties of the dirac matrices, we can considerly simplify the $|M|^2$ expression, since $k_1 = \gamma^{\mu} k_{1\mu}$, $k_2 = \gamma^{\nu} k_{2\nu}$ and $P_{jk} = factors * \gamma_5$

$$1)Tr[\gamma^{\mu}] = 0$$

$$2)Tr[\gamma^{\mu}\gamma_{5}] = 0$$

$$3)Tr[\gamma_{5}\gamma^{\mu}\gamma^{\nu}] = 0$$

$$4)\gamma_{5}^{2} = I_{4x4}$$

$$5)\gamma_{5}\gamma^{\mu} = -\gamma^{\mu}\gamma_{5}6)Tr[I_{4x4}] = 4$$
(4.45)

Consequently we obtain

$$|M_{jk}|^{2} = \frac{1}{4} |\alpha_{jk}|^{2} \left\{ [|S_{jk}|^{2} + |P_{jk}^{'}|^{2}] Tr[k_{2}k_{1}] + 4[|P_{jk}^{'}|^{2} - |S_{jk}|^{2}]m_{\tau}m_{\mu} \right\}$$
(4.46)

where P'_{jk} is the scalar part of P_{jk} . In order to find the traces in the above expression, we use

$$Tr[k_{2}k_{1}] = g^{\mu\nu}k_{2\mu}k_{1\nu}$$

= $4k_{2} \cdot k_{1}$
= $4k_{2}^{\mu}k_{1\mu}$
= $4(\frac{E_{2}}{c}, \vec{k}_{2}) \cdot (\frac{E_{1}}{c}, -\vec{k}_{1})$
(4.47)

If we take as our reference the particle h^0 , the momentum conservation leads us to

$$\vec{p}_{h^0} = \vec{k}_1 + \vec{k}_2 = 0$$

$$\vec{k}_1 = -\vec{k}_2$$
(4.48)

Then

$$Tr[k_2k_1] = 4\{\frac{E_2E_1}{c^2} + |\vec{k}_2|^2\}$$
(4.49)

where we labed E_T as the total energy, and used the conservation of Energy. We re-express $|k_2|$

$$|k_{2}|^{2} = |k_{3}|^{2}$$

$$\Longrightarrow$$

$$\frac{E_{2}^{2}}{c^{2}} - m_{2}^{2}c^{2} = \frac{E_{3}^{2}}{c^{2}} - m_{3}^{2}c^{2}$$

$$\Longrightarrow$$

$$E_{2}^{2} - E_{3}^{2} = m_{2}^{2}c^{4} - m_{3}^{2}c^{4}$$

$$(E_{2} + E_{3})(E_{2} - E_{3}) = m_{2}^{2}c^{4} - m_{3}^{2}c^{4}$$

$$(4.50)$$

If we use

$$E_T = m_{h^\circ}c^2 = E_2 + E_3$$

$$\Longrightarrow$$

$$m_{h^\circ}c^2 - 2E_3 = E_2 - E_3$$
(4.51)

We obtain

$$m_{h^0}c^2(m_{h^0}c^2 - 2E_3) = m_2^2c^4 - m_3^2c^4$$
(4.52)

Substituing E_3 and isolating $|k_2|$ we have

$$\rho = \frac{\sqrt{C_{h^0\mu\tau}}}{2m_{h^0}} \tag{4.53}$$

where we just renamed $|k_2|$ as ρ . For our special case, all the labed momentums, masses and Energies with the number 2 will be referenced to the τ and with the number 1 to the μ

42

Taking $c = 1, \bar{h} = 1$, the expression $|M|^2$ is given by

$$|M_{jk}|^{2} = |\alpha_{jk}|^{2} \left\{ \left(|S_{jk}|^{2} + |P_{jk}^{'}|^{2} \right) \left(E_{\tau} E_{\mu} + \rho^{2} \right) + \left(|P_{jk}^{'}|^{2} - |S_{jk}|^{2} \right) m_{\tau} m_{\mu} \right\}$$

$$(4.54)$$

We have that

$$\Gamma(h^0 - > \mu\tau) = \sum_{jk} \frac{1}{8\pi\hbar m_{h^0}} \int_{(m_\tau + m_\mu)c^2} |M_{jk}|^2 \frac{\delta(m_{h^0}c - \frac{E_T}{c})\rho}{E_T} dE_T \quad (4.55)$$

Substituing $|M|^2$, we obtain.

$$\Gamma(h^{0} - > \mu\tau) = \sum_{jk} \frac{c}{8\pi^{2}\hbar m_{h^{0}}} |\alpha_{jk}|^{2} \left\{ \left(|S_{jk}|^{2} + |P_{jk}^{'}|^{2} \right) \left(E_{\tau}E_{\mu} + \rho^{2} \right) \right. \\ \left. + \left(|P_{jk}^{'}|^{2} - |S_{jk}|^{2} \right) m_{\tau}m_{\mu} \right\} \int_{(m_{\tau} + m_{\mu})c^{2}} \frac{\delta(E_{T} - m_{h^{0}}c^{2})\rho}{E_{T}} dE_{T}$$

$$(4.56)$$

Using the function delta property

$$\int f(x)\delta(x-x')dx = f(x')$$
(4.57)

$$\Gamma(h^{0} - > \mu\tau) = \sum_{jk} \frac{|\alpha_{jk}|^{2} \rho}{8\pi^{2} \hbar m_{h^{0}}^{2} c} \bigg\{ \big(|S_{jk}|^{2} + |P_{jk}^{'}|^{2} \big) \big(E_{\tau} E_{\mu} + \rho^{2} \big) + [|P_{jk}^{'}|^{2} - |S_{jk}|^{2}] m_{\tau} m_{\mu} \bigg\}$$

$$(4.58)$$

Once again taking $c = 1, \bar{h} = 1$.

$$\Gamma(h^{0} - > \mu\tau) = \sum_{jk} \frac{|\alpha_{jk}|^{2} \rho}{8\pi^{2} m_{h^{0}}^{2}} \bigg\{ |S_{jk}|^{2} (E_{\tau} E_{\mu} + \rho^{2} - m_{\tau} m_{\mu}) + |P_{jk}^{'}|^{2} (E_{\tau} E_{\mu} + \rho^{2} + m_{\tau} m_{\mu}) \bigg\}$$

$$(4.59)$$

where

$$E_{\mu} = \sqrt{m_{\mu}^2 + \rho^2}$$
(4.60)

$$E_{\tau} = \sqrt{m_{\tau}^2 + \rho^2} \tag{4.61}$$

We divide the expression 4.59 by the total width. We used the value of 4 [MeV], which is between the range $6.1^{+7.7}_{-2.9}$ [MeV] [26] and in good accuracy with different papers where the total width is given [27], [28], [29], [30].

$\tilde{f}\tilde{f}$	$S_{jk} [1/\text{GeV}]$	$P_{jk} [1/\text{GeV}]$
$\tilde{\mu}_1 \tilde{\mu_1}$	$-8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk}-F_{c0}[C_{jk}+C_{h^0\mu\tau}(\frac{10}{8}m_{\tilde{B}}+m_{\tau})]\}$	$6i\pi^2 m_{ ilde{B}} F_{c0} \gamma_5$
$\tilde{\mu}_1 \tilde{\mu_2}$	$6i\pi^2 m_{ ilde{B}} F_{C0}$	$8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8}m_{\tilde{B}}]\}\gamma_5$
$\tilde{\mu}_1 \tilde{\tau}_1$	0	0
$\tilde{\mu}_1 \tilde{\tau}_2$	$-8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk}-F_{c0}[C_{jk}+C_{h^0\mu\tau}(m_\tau+\frac{10}{8}m_{\tilde{B}})]\}$	$6i\pi^2 F_{c0} m_{\tilde{B}} \gamma_5$
$\tilde{\mu}_2 \tilde{\mu}_1$	$6i\pi^2 m_{\tilde{B}}F_{c0}$	$-8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk}-F_{c0}[C_{jk}+C_{h^0\mu\tau}(\frac{10}{8}m_{\tilde{B}}+m_{\tau})]\}$
$\tilde{\mu}_2 \tilde{\mu}_2$	$8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8}m_{\tilde{B}})]\}$	$6i\pi^2 F_{c0} m_{\tilde{B}} \gamma_5$
$\tilde{\mu}_2 \tilde{\tau}_1$	$8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8}m_{\tilde{B}})]\}$	$6i\pi^2 F_{c0} m_{\tilde{B}} \gamma_5$
$\tilde{\mu}_2 \tilde{\tau}_2$	0	0
$ ilde{ au}_1 ilde{\mu}_1$	0	0
$\tilde{ au}_1 \tilde{\mu}_2$	$8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8}m_{\tilde{B}})]\}$	$6i\pi^2 F_{c0} m_{\tilde{B}} \gamma_5$
$\tilde{\tau}_1 \tilde{\tau}_1$	$8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8}m_{\tilde{B}})]\}$	$6i\pi^2 F_{c0} m_{ ilde{B}} \gamma_5$
$\tilde{\tau}_1 \tilde{\tau}_2$	$6i\pi^2 m_{ ilde{B}} F_{c0}$	$-8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk}-F_{c0}[C_{jk}+C_{h^0\mu\tau}(\frac{10}{8}m_{\tilde{B}}+m_{\tau})]\}$
$ ilde{ au}_2 ilde{\mu}_1$	$-8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk}-F_{c0}[C_{jk}+C_{h^0\mu\tau}(m_{\tau}+\frac{10}{8}m_{\tilde{B}})]\}$	$6i\pi^2 F_{c0} m_{\tilde{B}} \gamma_5$
$ ilde{ au}_2 ilde{\mu}_2$	0	0
$\tilde{\tau}_2 \tilde{\tau}_1$	$6i\pi^2 m_{ ilde{B}} F_{C0}$	$8\frac{i\pi^2}{C_{h^0\mu\tau}}\{\mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8}m_{\tilde{B}}]\}\gamma_5$
$\tilde{\tau}_2 \tilde{\tau}_2$	$\left -8\frac{i\pi^2}{C_{h^0\mu\tau}} \{ \mathcal{B}_{jk} - F_{c0}[C_{jk} + C_{h^0\mu\tau}(\frac{10}{8}m_{\tilde{B}} + m_{\tau})] \} \right $	$6i\pi^2 m_{\tilde{B}} F_{c0} \gamma_5$

Table 4.1: It is shown the Scalar and Pseudoscalar parts of M_{jk} . Units in $\frac{1}{GeV}$

\tilde{f} \tilde{f}	$\alpha_{jk} \; [\text{GeV}]$
$\tilde{\mu}_1 \tilde{\mu}_1$	$-g_{h^0\tilde{\mu}_1\tilde{\mu}_1}\frac{ig^2s_{\varphi}c_{\varphi}}{16}tan^2\theta_w$
$\tilde{\mu}_1 \tilde{\mu}_2$	$-g_{h^0\tilde{\mu}_1\tilde{\mu}_2}\frac{ig^2c_{\varphi}s_{\varphi}}{16}tan^2\theta_w$
$\tilde{\mu}_1 \tilde{\tau}_1$	0
$\tilde{\mu}_1 \tilde{\tau}_2$	$-g_{h^0\tilde{\mu}_1\tau_2}\frac{ig^2c_{\varphi}^2}{16}tan^2\theta_w$
$\tilde{\mu}_2 \tilde{\mu}_1$	$-g_{h^0\mu_2\mu_1}\frac{ig^2c_{\varphi}s_{\varphi}}{16}tan^2\theta_w$
$\tilde{\mu}_2 \tilde{\mu}_2$	$-g_{h^0\mu_2\mu_2}\frac{ig^2c_{\varphi}s\varphi}{16}tan^2\theta_w$
$\tilde{\mu}_2 \tilde{\tau}_1$	$-g_{h^0\tilde{\mu}_2\tilde{\tau}_1}\frac{ig^2c_{\varphi}^2}{16}tan^2\theta_w$
$\tilde{\mu}_2 \tilde{ au}_2$	0
$\tilde{ au}_1 \tilde{\mu}_1$	0
$ ilde{ au_1} ilde{\mu}_2$	$g_{h^0\tau_1\mu_2}rac{ig^2s^2\varphi}{16}tan^2 heta_w$
$\tilde{\tau}_1 \tilde{\tau}_1$	$-g_{h^0\tilde{\tau}_1\tilde{\tau}_1}\frac{ig^2c_{\varphi}s_{\varphi}}{16}tan^2\theta_w$
$\tilde{\tau}_1 \tilde{\tau}_2$	$g_{h^0\tilde{\tau}_1\tilde{\tau}_2}\frac{ig^2c_{\varphi}s_{\varphi}}{16}tan^2\theta_w$
$\tilde{ au}_2 \tilde{\mu}_1$	$-g_{h^0\tilde{\tau}_2\tilde{\mu}_1}\frac{ig^2s_{\varphi}^2}{16}tan^2\theta_w$
$\tilde{ au}_2 \tilde{\mu}_2$	0
$\tilde{\tau}_2 \tilde{\tau}_1$	$-g_{h^0\tilde{\tau_2}\tilde{\tau}_1}\frac{ig^2s_{\varphi}c_{\varphi}}{16}tan^2\theta_w$
$\tilde{ au}_2 \tilde{ au}_2$	$-g_{h^0\tilde{\tau}_2\tilde{\tau}_2}\frac{ig^2c_{\varphi}s_{\varphi}}{16}tan^2\theta_w$

Table 4.2: The respective coupling α_{jk} for each of the possible loop corrections. Units in GeV

Particle	Propagator	
\tilde{B}	$\frac{i({\not\!\! q}_2\!+\!m_{\tilde{B}})}{q_2^2\!-\!m_{\tilde{B}}^2}$	
$\tilde{\mu}_s$	$rac{i}{q_{1,3}^2 - m_{ ilde{\mu}_s}^2}$	
$ ilde{ au}_s$	$\frac{i}{q_{1,3}^2 - m_{\tilde{\tau}_s}^2}$	

Table 4.3: Expressions for the respective Propagators of the different particles, where n is the label of the momentum and it depends on how the diagram is constructed. j can be either 1 or 2 and are the different possible s-muons and s-taus

4.1.1 Calculations and Computing the Branching Ratio

We used FORTRAN to calculate the branching ratio. Since we have five free parameters in our Branching Ratio expression, we made the analysis taking into account randomly all the possible values that could take the parameters in certain regions. These regions are constrains of the experiments or there would be no theoretical reason to extend the variables to bigger or lower regions. The regions are given by

- μ_{susy} is a parameter that experimentally must be bigger than |500| GeV. Therefore we make plots variating this parameter randomly with positive values $\mu_{susy} > 500$ and negative values $\mu_{susy} < -500$. In figure $4.2(\mu_{susy} < 0)$ and in figure $4.7(\mu_{susy} > 0)$ are shown the randomly ploting of the Branching Ratio vs the free parameter μ_{susy} .
- m_0 is the a paramter that comes from the trilinear term in our Ansatz for Flavour Violation. It is the parameter for all the masses and it must be $m_0 \gtrsim 500 GeV$. If the value of m0 is lower than 500 GeV, the masses tend to be lower than 300 GeV, and if that were the case, the s-particle would have been already discovered. We take as a maximal value 5000 GeV for the same reason as the parameter A0. In figure $4.3(\mu_{susy} > 0)$ and in figure $4.8(\mu_{susy} < 0)$ are shown the randomly ploting of the Branching Ratio vs the free parameter m_0 .
- In figure $4.4(\mu_{susy} < 0)$ and in figure $4.9(\mu_{susy} > 0)$ are shown the randomly ploting of the Branching Ratio vs the free parameter $tan(\beta)$. $1 \leq tan(\beta) \leq 60$ since values lower to 1 tend to diverge to infinity. [11] Values of $tan(\beta)$ can not be bigger than 60, since it is the ratio of the vacuum spectation values of the Higgs Boson.
- A0 is measured in GeV and the value of the $A0 \approx 1000$. Therefore we took randomly possible values in the range 800 < A0 < 1200GeV. The value could be bigger. There is no experimental constraint. However, the value of the s-masses depend of A0 and values can make the s-particle masses tend to infinity which would have no physical meaning. In figure $4.5(\mu_{susy} < 0)$ and in figure $4.10(\mu_{susy} > 0)$ are shown the randomly ploting of the Branching Ratio vs the free parameter A_0 .
- $m_{\tilde{B}}$ can take any value greater than 500 GeV. It can not be lower since it would have been already discovered at experiments. We make the range of the bino mass from 500 to 5000 GeV. In figure $4.6(\mu_{susy} < 0)$ and in figure $4.11(\mu_{susy} > 0)$ are shown the randomly ploting of the Branching Ratio vs the free parameter $m_{\tilde{B}}$.



Figure 4.2: Plot of Branching Ratio and μ_{susy} , for $\mu_{susy} < 0$. All the values of $A0, m_0, tan(\beta)$, m_b are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.3: Plot of Branching Ratio and m_0 with $\mu_{susy} < 0$. All the values of $A0, \mu_{susy}, tan(\beta), m_b$ are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.4: Plot of Branching Ratio and $tan(\beta)$ with $\mu_{susy} < 0$. All the values of $A0, m_0, \mu_{susy}, m_b$ are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.5: Plot of Branching Ratio and A0 with $\mu_{susy} < 0$. All the values of $\mu_{susy}, m_0, tan(\beta), m_b$ are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.6: Plot of Branching Ratio and mb with $\mu_{susy} < 0$. All the values of $\mu_{susy}, m_0, tan(\beta)$, A0 are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.7: Plot of Branching Ratio and μ_{susy} , for $\mu_{susy} > 0$. All the values of $A0, m_0, tan(\beta)$, m_b are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.8: Plot of Branching Ratio and μ_0 with $\mu_{susy} > 0$. All the values of $A0, \mu_{susy}, tan(\beta), m_b$ are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.9: Plot of Branching Ratio and $tan(\beta)$ with $\mu_{susy} > 0$. All the values of $A0, m_0, \mu_{susy}, m_b$ are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.10: Plot of Branching Ratio and A0 with $\mu_{susy} > 0$. All the values of $\mu_{susy}, m_0, tan(\beta), m_b$ are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.



Figure 4.11: Plot of Branching Ratio and mb with $\mu_{susy} > 0$. All the values of $\mu_{susy}, m_0, tan(\beta)$, A0 are variated. The three lines represent the value of the best fit of the Branching Ratio found by CMS and the lower and upper limits of error to the measure. The green region represents the solutions of our Ansatz to the region given by CMS.