

## Chapter 3

# Flavour Violation within the Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model(MSSM) is a Quantum Field Theory which extends the Standard Model. This theory annexes a new symmetry, this is reason for the name of the theory. In theoretical physics we name a *symmetry* the transformation that leaves invariant the equations of the interactions. Example of it is the Electromagnetic Equations of Maxwell which are invariant under the *Lorentz* transformations. In the case of the Minimal Supersymmetric Standard Model the new symmetry transforms the fermionic fields and the bosonic fields into one another. As an implication of this transformations we have that every elementary particle mentioned in chapter 2 have a *super-partner*. Each superpartner particle differs of  $\frac{1}{2}$  of spin. For example the sleptons will be spin 0 particles. The Higgsino which is the superpartner of the Higgs boson will be spin  $\frac{1}{2}$  particle. In figure 3.1 are shown the particles predicted in the Standard Model and their respective super-partners of the MSSM.

Supersymmetry(SUSY) has become a controversial theoretical model because many scientists consider it the correct model to extend the Standard Model. It is considered a very natural model that can explain dark matter, gravity, vanish the radiative divergences to the quantum corrections to the Higgs boson. However, other scientists consider it incorrect arguing that the *superparticles* should have already been discovered. It will be the experiments in the particle accelerators that will tell us the truth.

We calculate within the MSSM the branching ratio of the decay  $h^0 \rightarrow \tau\mu$  that recently was found in CMS Experiment in CERN [10]. We will use an Ansatz that mixes the second and third family of sleptons. We firstly show the basic features of any supersymmetric theory. We based on Ref. [23] for all explicit

derivations of Supersymmetry. After that, it will be shown the MSSM main characteristics. Finally we explain the Ansatz used for our calculations.

The Lagrangian of MSSM has two general terms. The first one  $\mathcal{L}_{MSSM}$  and  $\mathcal{L}_{soft}$ . The soft supersymmetry breaking Lagrangian  $\mathcal{L}_{soft}$  has a trilinear scalar couplin term called  $A_{jk}$  *trilinear coupling*. This thesis is based on the Ansatz proposed in [11]. In general, the work done in this thesis was modifying the MSSM interaction Lagrangians using the Ansatz in the trilinear coupling and calculate the Branching Ratio.

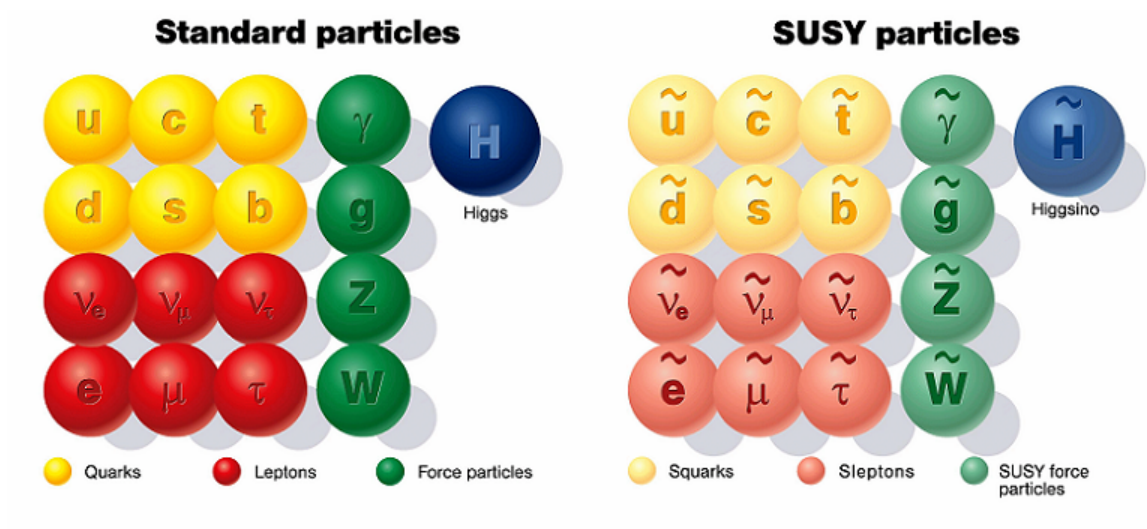


Figure 3.1: Standard Model Particles and their respective super-partner particle. [Illustration by CERN&SAR]

### 3.1 The Super-Poincaré Algebra

Any supersymmetric theory must satisfy the Super-Poincaré Algebra. Therefore the extension that we performed of the MSSM, satisfies the algebra mentioned. The Super-Poincaré Algebra extends the Poincaré Algebra where the Poincaré Algebra is defined by.

$$[P_\mu, P_\nu] = 0 \quad (3.1)$$

$$[M_{\mu\nu}, P_\rho] = -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu) \quad (3.2)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\rho}M_{\mu\sigma}) \quad (3.3)$$

where  $P_\mu$  is a generator of translation and  $M_{\mu\nu}$  is a generator of Lorentz transformation. In appendix C is shown the matrix representation of  $\eta_{\mu\rho}, M_{\mu\nu}$  The

extension should accomplish with the following algebra

$$[P_\mu, Q_a] = 0 \quad (3.4)$$

$$[M_{\mu\nu}, Q_a] = -(\sigma_{\mu\nu}^4)_{ab} Q_b \quad (3.5)$$

$$\{Q_a, \bar{Q}_b\} = 2(\gamma^\mu)_{ab} P_\mu \quad (3.6)$$

$$\{Q_a, Q_b\} = -2(\gamma^\mu C)_{ab} P_\mu \quad (3.7)$$

$$\{\bar{Q}_a, \bar{Q}_b\} = 2(C^{-1}\gamma^\mu)_{ab} P_\mu \quad (3.8)$$

where  $\sigma_{\mu\nu}^4 = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$ ,  $Q_a$  are 4 spinor charges generators,  
 $C = \eta_c \gamma^2 \gamma^0 = \eta_c \begin{pmatrix} -\sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$ ,  $C^{-1} = \eta_c^* \begin{pmatrix} -\sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$  and  $a, b \in \{1, 2, 3, 4\}$

## 3.2 The Wess Zumino Model

The Wess Zumino Model is the most basic SUSY Model that exists. It is not a realistic model, however it is an excellent starting point to understand supersymmetric models. The supersymmetric transformation is a transformation from bosons to fermions and viceversa. The Lagrangian proposed is

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu A)(\partial^\mu A) - \frac{1}{2}m^2 A^2 + \frac{1}{2}(\partial_\mu B)(\partial^\mu B) - \frac{1}{2}m^2 B^2 + \frac{1}{2}\bar{\Psi}(i\not{\partial} - m)\Psi \\ & - mgA(A^2 + B^2) - g(\bar{\Psi}\Psi A + i\bar{\Psi}\gamma^5\Psi B) - \frac{1}{2}g^2(A^2 + B^2)^2 \end{aligned} \quad (3.9)$$

with  $A = A^\dagger$ ,  $B = B^\dagger$  and  $\Psi = C\bar{\Psi}^T$ , where C is the charge conjugation matrix  $\begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}$  and  $\sigma^2$  is the Pauli matrix. “A” is a scalar field. A scalar field associates to each point of the space a scalar value. In particle physics the scalar fields are associated to spin 0 particles such as the Higgs Boson. B is a pseudo-scalar field, where the pseudoscalars change their sign under parity transformations  $((x, y, z) \rightarrow (-x, -y, -z))$ . The slashed derivative, is the Feynman notation for  $\not{\partial}_\mu = \gamma^\mu \partial_\mu$ . The field  $\Psi$  is associated to majorana particles. A majorana particle is any particle that is its own antiparticle. In this model all fields have the same mass  $m$  and couple with the same strength  $g$ . All these assumptions are given by the model. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}(q, \dot{q}) \quad (3.10)$$

i.e. the Lagrangian is dependent of the generalized coordinates and its derivative with respect time. In quantum field theory the Lagrangian will depend on the fields.

$$\mathcal{L}(\phi_i, \partial_\mu \phi_i) \quad (3.11)$$

Here  $\phi_i$  includes scalar and pseudoscalar fermionic fields i.e.  $\phi \in A, B, \Psi, \bar{\Psi}$ . We apply the equations of motion can be obtained with the Euler-Lagrange equation for each of the fields.

$$\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_i)} = 0 \quad (3.12)$$

where  $\phi_i \in \{A, B, \Psi, \bar{\Psi}\}$ .

Let  $\phi = A$ . Therefore we obtain

$$\frac{\partial \mathcal{L}}{\partial A} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A)} = 0 \quad (3.13)$$

where from 3.9

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial A} &= -m^2 A - g\bar{\psi}\psi - mg(3A^2 + B^2) - 2g^2 A(A^2 + B^2), \\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu A)} &= \partial^\mu A \end{aligned} \quad (3.14)$$

If we substitute eq. (3.14) in eq. 3.13

$$(\square + m^2)A = -g[\bar{\Psi}\Psi + m(3A^2 + B^2) + 2gA(A^2 + B^2)] \quad (3.15)$$

For the pseudoscalar field  $B$  we have that.

$$\frac{\partial \mathcal{L}}{\partial B} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu B)} = 0 \quad (3.16)$$

where

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial B} &= -m^2 B - ig\bar{\Psi}\gamma^5\Psi - 2mgAB - 2g^2 B(A^2 + B^2), \\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu B)} &= \partial^\mu B \end{aligned} \quad (3.17)$$

Hence we obtain that eq.(3.16) is

$$(\square + m^2)B = -g[i\bar{\Psi}\gamma^5\Psi + 2mAB + 2gB(A^2 + B^2)] \quad (3.18)$$

For the antifermionic  $\phi = \bar{\Psi}$ , the Euler-Lagrange Equation becomes

$$\frac{\partial \mathcal{L}}{\partial \bar{\Psi}} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\Psi})} = 0 \quad (3.19)$$

where

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{\Psi}} &= \frac{1}{2}(i\cancel{\partial} - m)\Psi - g\Psi A - ig\gamma_5\Psi B, \\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\Psi})} &= 0 \end{aligned} \quad (3.20)$$

Therefore

$$(i\cancel{\partial} - m)\Psi = 2g(A + i\gamma_5 B)\Psi \quad (3.21)$$

Finally, for the fermionic field  $\phi = \Psi$

$$\frac{\partial \mathcal{L}}{\partial \Psi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} = 0 \quad (3.22)$$

where

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \bar{\Psi}} &= -\frac{1}{2}m\bar{\Psi} - g\bar{\Psi}A - ig\bar{\Psi}\gamma_5 B, \\ \frac{\partial \mathcal{L}}{\partial \partial_\mu \bar{\Psi}} &= \frac{i}{2}\bar{\Psi}\gamma^\mu\end{aligned}\quad (3.23)$$

Hence

$$i(\not{\partial} + m)\bar{\Psi} = -2g(\bar{\Psi}A + i\bar{\Psi}\gamma_5 B) \quad (3.24)$$

Now, we are interested to include supersymmetric transformations. As it was mentioned before, the supersymmetric transformations will transform from a boson field to a fermion field. If these transformations leave the action invariant they can be considered as symmetries. The action is defined as usual.

$$S = \int \mathcal{L} d^4x \quad (3.25)$$

where  $\mathcal{L}$  is the Lagrangian density where and the variation of  $\mathcal{L}$  is a total derivative if the transformation  $\delta$  is a symmetry.

$$\mathcal{L}' - \mathcal{L} := \delta\mathcal{L} = \partial_\mu V^\mu \neq 0 \quad (3.26)$$

It follows from *Noether Theorem* that the invariance of the action under a symmetry transformation always implies the existence of a conserved current.

$$\partial_\mu j^\mu = 0 \quad (3.27)$$

where  $j^\mu$  would be the current. We are interested in finding a current which is invariant under supersymmetry transformation. The variation of the Lagrangian density under an arbitrary infinitesimal variation of the fields yields to

$$\begin{aligned}A \longrightarrow A' &= A + \delta A, \Rightarrow A' - A = \delta A \\ B \longrightarrow B' &= B + \delta B, \\ \Psi \longrightarrow \Psi' &= \Psi + \delta\Psi \\ \delta\mathcal{L} &= \mathcal{L}(\phi'_i, \partial_\mu \phi'_i) - \mathcal{L}(\phi_i, \partial_\mu \phi_i) \\ &= \frac{\partial \mathcal{L}}{\partial \phi_i} \delta\phi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\partial_\mu \phi_i \\ &= \frac{\partial \mathcal{L}}{\partial \phi_i} \delta\phi_i - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\phi_i + \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\phi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\partial_\mu \phi_i \text{ Summing Zero} \\ &= \left\{ \frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) \right\} \delta\phi_i + \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\phi_i \right) \\ &= \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\phi_i \right) \text{ We used Euler-Lagrange equation (3.12)}\end{aligned}\quad (3.28)$$

From equation (3.26) we get

$$\begin{aligned}\delta\mathcal{L} &\equiv \partial_\mu V^\mu \implies \partial_\mu \left( V^\mu - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\phi_i \right) = 0 \\ \therefore j^\mu &= V^\mu - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta\phi_i \text{ Using Eq.(3.27)}\end{aligned}\quad (3.29)$$

where we obtained the definition of the current to express the conservation equation.

We propose the symmetries of the fields which transform fermions and bosons into each other and we find the expression for the current  $j^\mu$ . After we show that the action is invariant, we can conclude that this is a theory with a new supersymmetric symmetry, which is called the Wess-Zumino Model. The following variations for the different fields represent a supersymmetry transformation i.e. *bosons*  $\Leftrightarrow$  *fermions*

$$\begin{aligned}
\delta A &= \bar{\epsilon}\Psi(x) \\
\delta B &= -i\bar{\epsilon}\gamma^5\Psi(x) \\
\delta\Psi &= -(i\cancel{\partial} + m)(A - i\gamma^5 B)\epsilon \\
\delta\bar{\Psi} &= \bar{\epsilon}(A - i\gamma^5 B)(i\overleftarrow{\cancel{\partial}} - m)
\end{aligned} \tag{3.30}$$

where  $\epsilon$  is an independent Grassman variable. The equations above constitute the infinitesimal supersymmetry transformation of the fields  $A, B$  and  $\Psi$ . We have now the properties to compute  $\delta\mathcal{L}$ .

For simplicity we consider only the free part of the Lagrangian  $\mathcal{L}_{free} = \mathcal{L}|_{g=0}$

$$\mathcal{L}_{free} = \frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}m^2 A^2 + \frac{1}{2}(\partial_\mu B)^2 - \frac{1}{2}m^2 B^2 + \frac{1}{2}\bar{\Psi}(i\cancel{\partial} - m)\Psi \tag{3.31}$$

Using Eq.(3.30)

$$\begin{aligned}
\delta\mathcal{L}_{free} &= (\partial_\mu A)\delta(\partial^\mu A) - m^2 A\delta A + (\partial_\mu B)\delta(\partial^\mu B) - m^2 B\delta B + \frac{1}{2}\delta\bar{\Psi}(i\cancel{\partial} - m)\Psi + \frac{1}{2}\bar{\Psi}(i\cancel{\partial} - m)\delta\Psi \\
&= \bar{\epsilon}(\partial_\mu A)(\partial^\mu\Psi(x)) - m^2 A\bar{\epsilon}\Psi(x) - i\bar{\epsilon}\gamma^5(\partial_\mu B)\partial^\mu\Psi(x) + i\bar{\epsilon}\gamma^5 m^2 B\Psi(x) \\
&+ \frac{1}{2}\bar{\epsilon}(A - i\gamma^5 B)(i\overleftarrow{\cancel{\partial}} - m)(i\cancel{\partial} - m)\Psi - \frac{1}{2}\bar{\Psi}(i\cancel{\partial} - m)(i\cancel{\partial} + m)(A - i\gamma^5 B)\epsilon \\
&= \bar{\epsilon}(\partial_\mu A)(\partial^\mu\Psi(x)) - m^2 A\bar{\epsilon}\Psi(x) - i\bar{\epsilon}\gamma^5(\partial_\mu B)(\partial^\mu\Psi(x)) + i\bar{\epsilon}\gamma^5 m^2 B\Psi(x) \\
&+ \frac{1}{2}\bar{\Psi}(\square + m^2)(A - i\gamma^5 B)\epsilon \text{ Using Eq.(3.21)for } \mathcal{L}|_{free}
\end{aligned} \tag{3.32}$$

where

$$\cancel{\partial}\cancel{\partial} = \partial_\mu\gamma^\mu\partial_\nu\gamma^\nu = \frac{1}{2}\partial_\mu\partial_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = \partial_\mu\partial_\nu\eta^{\mu\nu} = \partial_\mu\partial^\mu = \square \tag{3.33}$$

However the last term in Eq. (3.32) vanishes. From eq. (3.18) we know that  $\square + m^2 = 0$  for  $\mathcal{L}|_{free}$

$$\begin{aligned}
\delta\mathcal{L}_{free} &= \bar{\epsilon}(\partial_\mu A)(\partial^\mu\Psi(x)) - m^2 A\bar{\epsilon}\Psi(x) - i\bar{\epsilon}\gamma^5(\partial_\mu B)(\partial^\mu\Psi(x)) + i\bar{\epsilon}\gamma^5 m^2 B\Psi(x) \\
&= \bar{\epsilon}(\partial_\mu A)(\partial^\mu\Psi(x)) + \square A\bar{\epsilon}\Psi(x) - i\bar{\epsilon}\gamma^5(\partial_\mu B)(\partial^\mu\Psi) - i\bar{\epsilon}\gamma^5 \square B\Psi(x) \\
&= \partial_\mu\{\bar{\epsilon}(\partial^\mu A)\Psi - i\bar{\epsilon}\gamma^5(\partial^\mu B)\Psi\} \\
&= \partial_\mu\{\bar{\epsilon}[\partial^\mu(A - i\gamma^5 B)\Psi]\} \\
&=: \partial_\mu V^\mu
\end{aligned} \tag{3.34}$$

Hence

$$V^\mu = \bar{\epsilon}[\partial^\mu(A - i\gamma^5 B)]\Psi \quad (3.35)$$

Now, we find the current defined in Eq.(3.29). We use equations (3.14), (3.18) and (3.23). First we find the value of the second term of 3.29

$$\begin{aligned} \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta \phi_i &= \frac{\partial \mathcal{L}}{\partial(\partial_\mu A)} \delta A + \frac{\partial \mathcal{L}}{\partial(\partial_\mu B)} \delta B + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} \delta \Psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\Psi})} \delta \bar{\Psi} \\ &= (\partial_\mu A) \delta A + (\partial_\mu B) \delta B + \frac{i}{2} \bar{\Psi} \gamma_\mu \delta \Psi \\ &= (\partial_\mu A) \bar{\epsilon} \Psi - i(\partial_\mu B) \bar{\epsilon} \gamma^5 \Psi - \frac{i}{2} \bar{\Psi} \gamma_\mu (i\overleftarrow{\not{\partial}} + m)(A - i\gamma^5 B) \epsilon \\ &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi - \frac{i}{2} \bar{\Psi} \gamma_\mu (i\overleftarrow{\not{\partial}} + m)(A - i\gamma^5 B) \epsilon \\ &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi + \frac{i}{2} \epsilon^T (A - i\gamma^5 B)^T (i\overleftarrow{\not{\partial}} + m)^T \gamma_\mu^T \bar{\Psi}^T \end{aligned} \quad (3.36)$$

where we used transpose properties for commuting factors as  $X = ([X]^T)^T$

$$\begin{aligned} \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta \phi_i &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi + \frac{i}{2} \epsilon^T C^{-1} C (A - i\gamma^5 B)^T (i\overleftarrow{\not{\partial}} + m)^T \gamma_\mu^T \bar{\Psi}^T \\ &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi - \frac{i}{2} \bar{\epsilon} (AC - iC(\gamma^5)^T B) (i\overleftarrow{\not{\partial}}_p(\gamma^p)^T + m) \gamma_\mu^T \bar{\Psi}^T \end{aligned} \quad (3.37)$$

where we introduced the identity matrix as  $I = CC^{-1}$ , where  $C$  is the charge conjugation matrix. In the second step, since  $A = A^\dagger$  and  $B = B^\dagger$ , there is no modification by the transpose operation.  $A$  and  $B$  can commute with the  $C$  matrix since they are scalar and pseudoscalar fields. Also  $\bar{\epsilon} = \epsilon^T C^{-1}$

$$\begin{aligned} \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta \phi_i &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi - \frac{i}{2} \bar{\epsilon} (AC - iC(\gamma^5)^T B) (i\overleftarrow{\not{\partial}}_p(\gamma^p)^T + m) \gamma_\mu^T \bar{\Psi}^T \\ &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi + \frac{i}{2} \bar{\epsilon} (A - i\gamma^5 B) (i\overleftarrow{\not{\partial}}_p C(\gamma^p)^T + mC) C^{-1} \gamma_\mu C \bar{\Psi}^T \\ &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi + \frac{i}{2} \bar{\epsilon} (A - i\gamma^5 B) (i\overleftarrow{\not{\partial}}_p C(\gamma^p)^T C^{-1} + m) \gamma_\mu C \bar{\Psi}^T \\ &= \bar{\epsilon}[\partial_\mu(A - i\gamma^5 B)]\Psi - \frac{i}{2} \bar{\epsilon} (A - i\gamma^5 B) (i\overleftarrow{\not{\partial}}_p \gamma^p - m) \gamma_\mu \Psi \end{aligned} \quad (3.38)$$

where we used  $(\gamma^\mu)^T = -C^{-1} \gamma^\mu C$  and  $\Psi = C \bar{\Psi}^T$  Therefore

$$\begin{aligned} j^\mu &= V^\mu - \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \delta \phi_i \\ &= \frac{i}{2} \bar{\epsilon} (A - i\gamma^5 B) (i\overleftarrow{\not{\partial}} - m) \gamma^\mu \Psi \end{aligned} \quad (3.39)$$

The current  $j^\mu$  found is called the *supercurrent* in Supersymmetry. The existence of it, by *Noether Theorem* affirms us that the transformations from bosons

to fermions that we defined leave the action invariant.

We will show that the supercurrent is a conserved current. We re-express the supercurrent by the relation

$$j^\mu = \frac{1}{\lambda} \bar{\epsilon} k^\mu \quad (3.40)$$

where  $\lambda$  is just a real constant. Thus in the Wess-Zumino model the spinor current density

$$\begin{aligned} k^\mu &= \frac{i}{2} \lambda (A - i\gamma^5 B) (i\overleftarrow{\not{\partial}} - m) \gamma^\mu \Psi \\ &= -\frac{i}{2} \lambda m \gamma^\mu (A + i\gamma^5 B) \Psi - \frac{1}{2} \lambda [\not{\partial} (A + i\gamma^5 B)] \gamma^\mu \Psi \end{aligned} \quad (3.41)$$

Verifying that the supercurrent is conserved for free Lagrangian

$$\begin{aligned} \partial_\mu k^\mu &= -\frac{i}{2} \lambda m \not{\partial} [(A + i\gamma^5 B) \Psi] - \frac{1}{2} \lambda [\not{\partial} \not{\partial} (A - i\gamma^5 B)] \Psi - \frac{1}{2} \lambda [\not{\partial} (A + i\gamma^5 B)] \not{\partial} \Psi \\ &= -\frac{i}{2} \lambda m (\not{\partial} A - i\gamma^5 \not{\partial} B) \Psi - \frac{i}{2} m \lambda (A - i\gamma^5 B) \not{\partial} \Psi \\ &\quad - \frac{1}{2} \lambda [\not{\partial} \not{\partial} A - i\gamma^5 \not{\partial} \not{\partial} B] \Psi - \frac{1}{2} \lambda [\not{\partial} (A + i\gamma^5 B)] \not{\partial} \Psi \\ &= -\frac{1}{2} \lambda (\not{\partial} A - i\gamma^5 \not{\partial} B) (\not{\partial} \Psi + im \Psi) - \frac{1}{2} \lambda [\partial_\mu \partial^\mu A - i\gamma^5 \partial_\mu \partial^\mu B] \Psi - \frac{i}{2} m \lambda (A - i\gamma^5 B) \not{\partial} \Psi \\ &= \frac{i}{2} \lambda (\not{\partial} A - i\gamma^5 \not{\partial} B) (i\overleftarrow{\not{\partial}} - m) \Psi - \frac{1}{2} \lambda [\square A - i\gamma^5 \square B] \Psi - \frac{1}{2} m^2 \lambda (A - i\gamma^5 B) \Psi \\ &= -\frac{1}{2} \lambda \{(\square + m^2) A - i\gamma^5 (\square + m^2) B\} \Psi \\ &= 0 \end{aligned} \quad (3.42)$$

where we used the anticommutation relation  $\{\gamma^\mu, \gamma^5\} = 0$ . Therefore the supercurrent is conserved which implies that there exists a symmetry with the transformations from bosons to fermions and viceversa. From Noether Theorem, it implies that the charge is the integral of the zero component of the zero component of the density current in the volume.

$$Q_a = \int d^3x k_a^0 \quad (3.43)$$

where from (3.41)

$$k_a^0 = \frac{i}{2} \lambda \left[ \{(A - i\gamma^5 B) (i\overleftarrow{\not{\partial}} - m)\} \gamma^0 \Psi \right]_a \quad (3.44)$$

It can be demonstrated that this supersymmetric charge satisfies the Super-Poincaré algebra

$$\{Q_a, Q_b\} = 2P_\mu (\gamma^\mu)_{ab} \quad (3.45)$$

Therefore we have found a complete supersymmetric model. It is important to understand this basic model, since other supersymmetric models are built in



similar way. We exposed this model since for the thesis purposes there would be no point in finding all the supersymmetric lagrangian of MSSM. The MSSM Lagrangian is extremely long and the goal of the thesis could be missed with so many calculations. Therefore, we just showed the general steps for finding a supersymmetric theory.

### 3.3 SuperFields and Supercoordinates

In the specific case of the Minkowski space, the Euclidian space of three dimensions is extended to a four dimensions one (space-time). The reason is that it is needed to have an space where theories can be constructed with Lorentz invariance. In the analogous case, the superspace is constructed for supersymmetric invariances.

The Minkowski space is extended with four more dimensions. The elements of the superspace are called *supercoordinates*. The elements consist of the four space-time coordinates and four anticommuting Grassman numbers. In terms of the two-component Weyl spinor formalism, the four extra terms are  $\{\theta_A\}_{A=1,2}$  and  $\{\bar{\theta}_{\dot{B}}\}_{\dot{B}=\dot{1},\dot{2}}$ . Both two-component Weyl spinors which transform under the self-representation of  $SL(2,C)$  and the complex conjugate self-representation of  $SL(2,C)$  respectively. The  $SL(2,C)$  is the group of invertible complex matrices with unit determinant and the Lorentz transformation group.

The anticommutation relations of the two-component Weyl spinors are

$$\{\theta_A, \theta_B\} = 0 \quad (3.46)$$

$$\{\bar{\theta}_{\dot{A}}, \bar{\theta}_{\dot{B}}\} = 0 \quad (3.47)$$

$$\{\theta_A, \bar{\theta}_{\dot{B}}\} = 0 \quad (3.48)$$

And every element of the superspace is denotaded by

$$(x_\mu, \theta_A, \bar{\theta}_{\dot{A}}) \quad (3.49)$$

The important closure relations of the Super-Poincaré Algebra

$$\begin{cases} \{Q_A, Q_B\} = 0 \\ \{\bar{Q}_{\dot{A}}, \bar{Q}_{\dot{B}}\} = 0 \\ \{Q_A, \bar{Q}_{\dot{B}}\} = 2\sigma_{A,\dot{B}}^\mu P_\mu \end{cases} \quad (3.50)$$

They can be rewritten as commutators with the super-coordinates

$$\begin{cases} [\theta^A Q_A, \theta^B Q_B] = 0 \\ [\bar{\theta}_{\dot{A}} \bar{Q}^{\dot{A}}, \bar{\theta}_{\dot{B}} \bar{Q}^{\dot{B}}] = 0 \\ [\theta^A Q_A, \bar{\theta}_{\dot{B}} \bar{Q}^{\dot{B}}] = 2\theta^A \sigma_{A,\dot{B}}^\mu \bar{\theta}^{\dot{B}} P_\mu \end{cases} \quad (3.51)$$

In section 3.2 we mentioned infinitesimal supersymmetric transformations that left the action invariant. The transformations can be extended to finite supersymmetric transformations. Therefore it can be constructed and defined the

following operator in terms of the super-coordinates

$$L(x_\mu, \theta_A, \bar{\theta}^{\dot{A}}) := \exp(-ix_\mu P^\mu + i\theta Q + i\bar{\theta}\bar{Q}) \quad (3.52)$$

The terms  $P_\mu$ ,  $Q_A$  and  $Q^{\dot{A}}$  are hermitian operators which act on functions in superspace. These operators correspond to the basic elements of the Super-Poincaré algebra.

A general superfield  $\Phi$  is an operator-valued function defined on superspace in terms of its power series expansion in  $\theta$  and  $\bar{\theta}$ . An expansion example would be

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= f(x) + \theta^A \phi_A(x) + \bar{\theta}_{\dot{A}} \bar{\chi}^{\dot{A}}(x) + (\theta\theta)m(x) + (\bar{\theta}\bar{\theta})n(x) \\ &+ (\theta\sigma^\mu\bar{\theta})V_\mu(x) + (\theta\theta)\bar{\theta}_{\dot{A}} \bar{\lambda}^{\dot{A}}(x) + (\bar{\theta}\bar{\theta})\theta^A \Psi_A(x) + (\theta\theta)(\bar{\theta}\bar{\theta})d(x) \end{aligned} \quad (3.53)$$

where  $f(x)$ ,  $\phi_A(x)$ ,  $\bar{\chi}^{\dot{A}}(x)$ ,  $n(x)$ ,  $m(x)$ ,  $V_\mu(x)$ ,  $\Psi_A(x)$ ,  $d(x)$ ,  $\bar{\lambda}^{\dot{A}}(x)$  are called component fields.  $(\theta\theta) \equiv \theta^A \theta_A$  and  $(\bar{\theta}\bar{\theta}) \equiv \bar{\theta}_{\dot{A}} \bar{\theta}^{\dot{A}}$ . In general it can be thought just as the sum of the required fields in order to construct the theory. The superfield must have all of the bosonic, fermionic, and auxiliary fields within the corresponding supermultiplet. Therefore we can establish that

$$\begin{aligned} f(x), m(x), n(x) &\longrightarrow \text{complex scalar or pseudoscalar fields (Bosons)} \\ \Psi(x), \phi(x) &\longrightarrow \text{left-handed Weyl spinor fields (fermions)} \\ \bar{\chi}(x), \bar{\lambda}(x) &\longrightarrow \text{right-handed Weyl spinor fields (fermions)} \\ V_\mu(x) &\longrightarrow \text{Lorentz four-vector field. (Any vectorial field as electromagnetic field)} \\ d(x) &\longrightarrow \text{scalar field (As an example it could be the Higgs Boson field)} \end{aligned} \quad (3.54)$$

And the infinitesimal transformation of a superfield would be

$$\begin{aligned} \delta_s \Phi &= T_\alpha \Phi(x, \theta, \bar{\theta}) - \Phi(x, \theta, \bar{\theta}) \\ &= \left\{ \alpha \frac{\partial}{\partial \theta} + \bar{\alpha} \frac{\partial}{\partial \bar{\theta}} + i(\theta\sigma^\mu \bar{\alpha} - \alpha\sigma^\mu \bar{\theta}) \partial_\mu + \dots \right\} \Phi(x, \theta, \bar{\theta}) \end{aligned} \quad (3.55)$$

where  $T_\alpha = \Phi(x + i\theta\sigma\bar{\alpha} - i\alpha\sigma\bar{\theta}, \theta + \alpha, \bar{\theta} + \bar{\alpha})$ . Similarly it can be thought the infinitesimal transformation as the transformation applied to each of the component fields.

$$\begin{aligned} \delta_s \Phi(x, \theta, \bar{\theta}) &= \delta'_s f(x) + \theta^A \delta'_s \phi_A(x) + \bar{\theta}_{\dot{A}} \delta'_s \bar{\chi}^{\dot{A}}(x) + (\theta\theta) \delta'_s m(x) + (\bar{\theta}\bar{\theta}) \delta'_s n(x) \\ &+ (\theta\sigma^\mu \bar{\theta}) \delta'_s V_\mu(x) + (\theta\theta) \bar{\theta}_{\dot{A}} \delta'_s \bar{\lambda}^{\dot{A}}(x) + (\bar{\theta}\bar{\theta}) \theta^A \delta'_s \Psi_A(x) + (\theta\theta)(\bar{\theta}\bar{\theta}) \delta'_s d(x) \end{aligned} \quad (3.56)$$

After many operations and manipulations of the properties of the superfields

and super-coordinates it would be found for this specific superfield that

$$\begin{aligned}
\delta_s \Phi(x, \theta, \bar{\theta}) &= \alpha \phi(x) + \bar{\alpha} \bar{\chi}(x) + \theta \{ 2\alpha m(x) + i(\sigma^\mu \bar{\alpha}) \partial_\mu f(x) + (\sigma^\mu \bar{\alpha}) V_\mu(x) \} \\
&+ \bar{\theta} \{ 2\bar{\alpha} n(x) + i(\alpha \sigma^\mu \epsilon) \partial_\mu f(x) - (\alpha \sigma^\mu \epsilon) V_\mu(x) \} + (\theta \theta) \{ \bar{\alpha} \bar{\lambda}(x) - \frac{i}{2} \partial_\mu \phi(x) \sigma^\mu \bar{\alpha} \} \\
&+ (\bar{\theta} \bar{\theta}) \{ \alpha \Psi(x) + \frac{i}{2} \alpha \sigma^\mu \partial_\mu \bar{\chi}(x) \} \\
&+ (\theta \sigma^\mu \bar{\theta}) \{ \alpha \sigma_\mu \bar{\lambda}(x) + \Psi(x) \sigma_\mu \bar{\alpha} + \frac{i}{2} \alpha \partial_\mu \phi(x) - \frac{i}{2} \partial_\mu \bar{\chi}(x) \bar{\alpha} \} \\
&+ (\theta \theta) \bar{\theta} \{ 2\bar{\alpha} d(x) + \frac{i}{2} \bar{\alpha} \partial^\mu V_\mu(x) + i(\alpha \sigma^\mu \epsilon) \partial_\mu m(x) \} \\
&+ (\bar{\theta} \bar{\theta}) \theta \{ 2\alpha d(x) - \frac{i}{2} \alpha \partial^\mu V_\mu(x) + i(\sigma^\mu \bar{\alpha}) \partial_\mu n(x) \} \\
&+ (\theta \theta) (\bar{\theta} \bar{\theta}) \frac{i}{2} \{ \partial_\mu \Psi(x) \sigma^\mu \bar{\alpha} + \alpha \sigma^\mu \partial_\mu \bar{\lambda}(x) \}
\end{aligned} \tag{3.57}$$

Comparing each of the coefficients of the same powers of  $\theta$  and  $\bar{\theta}$  in Eq. 3.56 and Eq. 3.57 we know the particular infinitesimal transformations of the component fields.

$$\delta'_s f(x) = \alpha \phi(x) + \bar{\alpha} \bar{\chi}(x) \tag{3.58}$$

$$\delta'_s \phi_A(x) = 2\alpha m(x) + i(\sigma^\mu \bar{\alpha}) \partial_\mu f(x) + (\sigma^\mu \bar{\alpha}) V_\mu(x) \tag{3.59}$$

$$\delta'_s \bar{X}^A(x) = 2\bar{\alpha} n(x) + i(\alpha \sigma^\mu \epsilon) \partial_\mu f(x) - (\alpha \sigma^\mu \epsilon) V_\mu(x) \tag{3.60}$$

$$\delta'_s m(x) = \bar{\alpha} \bar{\lambda}(x) - \frac{i}{2} \partial_\mu \phi(x) \sigma^\mu \bar{\alpha} \tag{3.61}$$

$$\delta'_s n(x) = \alpha \Psi(x) + \frac{i}{2} \alpha \sigma^\mu \partial_\mu \bar{\chi}(x) \tag{3.62}$$

$$\delta'_s V_\mu(x) = \alpha \sigma_\mu \bar{\lambda}(x) + \Psi(x) \sigma_\mu \bar{\alpha} + \frac{i}{2} \alpha \partial_\mu \phi(x) - \frac{i}{2} \partial_\mu \bar{\chi}(x) \bar{\alpha} \tag{3.63}$$

$$\delta'_s \bar{\lambda}^A(x) = 2\bar{\alpha} d(x) + \frac{i}{2} \bar{\alpha} \partial^\mu V_\mu(x) + i(\alpha \sigma^\mu \epsilon) \partial_\mu m(x) \tag{3.64}$$

$$\delta'_s \Psi_A(x) = 2\alpha d(x) - \frac{i}{2} \alpha \partial^\mu V_\mu(x) + i(\sigma^\mu \bar{\alpha}) \partial_\mu n(x) \tag{3.65}$$

$$\delta'_s d(x) = \frac{i}{2} \{ \partial_\mu \Psi(x) \sigma^\mu \bar{\alpha} + \alpha \sigma^\mu \partial_\mu \bar{\lambda}(x) \} \tag{3.66}$$

From the relations above we can observe that each bosonic field transforms into a fermionic field and viceversa. The supersymmetric transformations are established automatically if we use the superspace formalism. It is the great advantage of using the superspace formalism. We did not need to force or establish the relations from a first instance as we did with the Wess-Zumino Model, they came up naturally. The MSSM and other supersymmetric theories are constructed with the superspace formalism because it ensures automatically the supersymmetric transformations and the invariance of the action under supersymmetric transformations.

### 3.4 Soft Supersymmetry Breaking

If supersymmetry were true in any energy scale, the super-particles would have been already discovered in the experiments.<sup>1</sup> Therefore, if supersymmetry is correct there should exist a Lagrangian density that is invariant under supersymmetry, but a vacuum state that is not. To this mechanism is called as *spontaneous* supersymmetry breaking. If the vacuum state  $|0\rangle$  is not invariant under supersymmetry transformations, then

$$Q_\alpha |0\rangle \neq 0, Q_\alpha^\dagger |0\rangle \neq 0, \quad (3.67)$$

Furthermore, if supersymmetry is spontaneously broken in the vacuum state, then the vacuum must have positive energy

$$\langle 0|H|0\rangle = \frac{1}{4} \left( \|Q_1^\dagger |0\rangle\|^2 + \|Q_1 |0\rangle\|^2 + \|Q_2^\dagger |0\rangle\|^2 + \|Q_2 |0\rangle\|^2 \right) > 0 \quad (3.68)$$

With the considerations above, one can construct supersymmetric theories with this special process of supersymmetry breaking. Examples of spontaneous supersymmetry breaking are *The O’Raifeartaigh Model, Fayet-Iliopoulos, Extra-dimensional and anomaly-mediated supersymmetry*. However, we are interested in so called *soft* supersymmetry breaking. We introduce extra terms that break supersymmetry explicitly in the effective MSSM Lagrangian. The supersymmetry-breaking couplings should be soft (of positive mass dimension) in order to be able to naturally maintain a hierarchy between the electroweak scale and the Planck mass scale.

The soft supersymmetry breaking Lagrangian is composed of gaugino masses  $M_a$  for each gauge group, scalar squared-mass terms  $(m^2)_i^j$  and  $b_i^j$ , and 3 scalar couplings  $a^{ijk}$  and  $c_i^{jk}$ . The gaugino particles are the *gluino*, gravitino, the winos and the bino. Each of them are superpartner particles of a gauge field. We are interested in the bino particle, which is the superpartner of the U(1) gauge field which corresponds to the weak hypercharge. The soft terms are capable of giving masses to all of the scalars and gauginos in a theory. In next section the soft breaking lagrangian of MSSM is discussed. We are interested in the 3 scalar couplings, since the calculations that we perform are couplings of *higgs, sfermion, sfermion* where each of them is a scalar.

### 3.5 MSSM

The path to construct MSSM is similar to the Wess Zumino Model. Supersymmetric transformations are searched in order to make the action invariant and a super-current is found. As we mentioned, a supersymmetry break term should be annexed to the model and the model is constructed under the supercoordinates and superfields. The construction of MSSM is complicated and extensive. The goal of this chapter and in general of the thesis could be deviated if we attempt to deduce all the features of the MSSM. Therefore, we are not going to

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<sup>1</sup>Some published papers searching supersymmetry are cited in references [7], [8], [9]

extend all the construction of it. We will just focus in the soft supersymmetry breaking lagrangian. For more information about the construction of MSSM and extensive detailed calculations, we highly recommend [5], [6].

The Lagrangian that breaks supersymmetry softly in MSSM in a general format is given by

$$\mathcal{L}_{soft} = \mathcal{L}_{fermion}^{mass} + \mathcal{L}_{bino}^{mass} + \mathcal{L}_{wino}^{mass} + \mathcal{L}_{gluino}^{mass} + \mathcal{L}_{Higgsino} + \mathcal{L}_{h^0 \tilde{f}_j \tilde{f}_k} \quad (3.69)$$

In order to establish the free parameters of the model coming from this Lagrangian, we write down the form of the slepton masses and the Higgs- slepton- slepton couplings, the first and last term of eq. 3.69 , which are given as

$$\mathcal{L}_{soft}^{\tilde{l}} = -m_{\tilde{E}_{jk}}^2 \tilde{E}^j \tilde{E}^{k\dagger} - m_{\tilde{L}_{j,k}}^2 \tilde{L}^{j\dagger} \tilde{L}^k - (A_{e,jk} \tilde{E}^j \tilde{L}^k H_1 + h.c) \quad (3.70)$$

The term  $A_{e,jk}$  is called the trilinear term and  $L_j$  and  $E^J$  are the doublet and singlet slepton fields. In this term we include the flavour violation between the third and the second family.

### 3.6 MSSM Extended in Flavour Ansatz

We used the proposal given by Professor Melina Gómez Bock. [11]. It is an ansatz that mixes the second and the third family or generation of s-leptons. This mixture is able since any scalar particle with the same quantum numbers can mix through the soft SUSY parameters [24]. The sleptons have spin 0 since every sparticle differs of  $\frac{1}{2}$  to the Standard Model Particles. Consequently they are scalars and no constraint in mixing exist. Therefore we can write a so called mass matrix.

$$\tilde{M}_l^2 = \begin{pmatrix} m_{\tilde{e}_{LL}} & m_{\tilde{e}_{LR}} & 0 & 0 & 0 & 0 \\ m_{\tilde{e}_{LR}}^\dagger & m_{\tilde{e}_{RR}} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{\tilde{\mu}_{LL}} & m_{\tilde{\mu}_{LR}} & 0 & 0 \\ 0 & 0 & m_{\tilde{\mu}_{LR}}^\dagger & m_{\tilde{\mu}_{RR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{\tilde{\tau}_{LL}} & m_{\tilde{\tau}_{LR}} \\ 0 & 0 & 0 & 0 & m_{\tilde{e}_{LR}}^\dagger & m_{\tilde{e}_{RR}} \end{pmatrix} \quad (3.71)$$

It can be written as block matrices of  $3 \times 3$  as

$$\tilde{M}_f^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^{2\dagger} & M_{RR}^2 \end{pmatrix} \quad (3.72)$$

The  $M_{LL}$  called the *left-left* block,  $M_{LR}$  called the *left-right*,  $M_{RR}$  called the *right-right*, where Each block contains the terms of mass. The lepton-flavor conservation is violated by the non-vanishing off-diagonal elements of each matrix, and the size of such elements is strongly constrained from experiments. Therefore the Ansatz for flavour violation comes directly from the mass matrix. In the Ansatz is assumed that exist a degeneracy in the Left-Left and Right Right blocks of the matrix. Therefore

$$M_{LL}^2 \simeq M_{RR}^2 \simeq \tilde{m}_0^2 \mathbf{I}_{3 \times 3}, \quad (3.73)$$

The first family, that is  $e_L$  and  $e_R$ , has practically no mixing with the other two families since current data allow considerable mixing between the second and third slepton families but high suppression to the first family [13]. Therefore in the Ansatz is considered that the mixing is so small that we can despreciate it and take as zero.

$$A'_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & w & z \\ 0 & y & 1 \end{pmatrix} A_0, \quad (3.74)$$

The variables  $w, y$  and  $z$  can take values in the range  $[-1, 1]$ . The dominant terms give a  $4 \times 4$  decoupled block mass matrix, in the basis  $\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R$ . The mass matrix proposed is

$$\tilde{M}_l^2 = \left( \begin{array}{cc|cccc} m_0^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_0^2 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & m_0^2 & X_2 & 0 & A_z \\ 0 & 0 & X_2 & m_0^2 & A_y & 0 \\ 0 & 0 & 0 & A_y & m_0^2 & X_3 \\ 0 & 0 & A_z & 0 & X_3 & m_0^2 \end{array} \right), \quad (3.75)$$

with  $X_3 = \frac{1}{\sqrt{2}} A_0 v \cos \beta - \mu m_\tau \tan \beta$  and  $X_2 = A_w - \mu m_\mu \tan \beta$ . Where  $\mu$  is the  $SU(2)$  – *invariant* coupling of two different Higgs superfield doublets,  $A_0$  is the trilinear coupling scale and  $\tan \beta = \frac{v_2}{v_1}$  is the ratio of the two vacuum expectation values coming from the two neutral Higgs fields, these three are MSSM parameters [24, 25]. The part of the matrix of our interest will be the right inferior part, which will be a 4x4 matrix that includes the flavour violation of the second and third family. Therefore we despreciate the first two rows and columns of the matrix above.

$A_z = \frac{1}{\sqrt{2}} z A_0 v \cos \beta$ $A_y = \frac{1}{\sqrt{2}} y A_0 v \cos \beta$ $A_w = \frac{1}{\sqrt{2}} w A_0 v \cos \beta$
---

Table 3.1: *Explicit terms of the sfermion mass matrix ansatz.*

In order to obtain the physical slepton eigenstates, the  $4 \times 4$  mass sub-matrix is diagonalized given in (3.75). The physical slepton are the values that could be measured in a experiment. For simplicity the ansatz considers that  $z = y$ , which represent that the mixtures  $\tilde{\mu}_L \tilde{\tau}_R$  and  $\tilde{\mu}_R \tilde{\tau}_L$  are of the same order . Remembering that any matrix that is diagonalizable, we make a transformation as follows to diagonalize it

$$\tilde{M}_{Diag}^2 = Z_l M_l^2 Z_l^\dagger \quad (3.76)$$

where

$$M_l^2 = \begin{pmatrix} \tilde{m}_0^2 & X_2 & 0 & A_y \\ X_2 & \tilde{m}_0^2 & A_y & 0 \\ 0 & A_y & \tilde{m}_0^2 & X_3 \\ A_y & 0 & X_3 & \tilde{m}_0^2 \end{pmatrix}. \quad (3.77)$$

The matrix above is the  $4 \times 4$  that has the slepton mixing in the right-inferior part of 3.75.

After we make the transformation in 3.76 , we obtain that  $Z_l$  is given by

$$\begin{pmatrix} \tilde{\mu}_L \\ \tilde{\tau}_L \\ \tilde{\mu}_R \\ \tilde{\tau}_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin \frac{\varphi}{2} & -\cos \frac{\varphi}{2} & \sin \frac{\varphi}{2} & \cos \frac{\varphi}{2} \\ \cos \frac{\varphi}{2} & -\sin \frac{\varphi}{2} & -\cos \frac{\varphi}{2} & \sin \frac{\varphi}{2} \\ -\sin \frac{\varphi}{2} & \cos \frac{\varphi}{2} & -\sin \frac{\varphi}{2} & \cos \frac{\varphi}{2} \\ \cos \frac{\varphi}{2} & \sin \frac{\varphi}{2} & \cos \frac{\varphi}{2} & \sin \frac{\varphi}{2} \end{pmatrix} \begin{pmatrix} \tilde{\mu}_1 \\ \tilde{\tau}_1 \\ \tilde{\mu}_2 \\ \tilde{\tau}_2 \end{pmatrix} = Z_l, \quad (3.78)$$

with

$$\begin{aligned} \sin \varphi &= \frac{2A_y}{\sqrt{4A_y^2 + (X_2 - X_3)^2}}, \\ \cos \varphi &= \frac{(X_2 - X_3)}{\sqrt{4A_y^2 + (X_2 - X_3)^2}} \end{aligned} \quad (3.79)$$

The reason of the equate in 3.78 is that the equation give us the physical eigenstates. When we diagonalize the mass matrix  $M_l^2$  we obtain the following eigenvalues.

$$\begin{aligned} m_{\mu_1}^2 &= \frac{1}{2}(2\tilde{m}_0^2 + X_2 + X_3 - R), \\ m_{\mu_2}^2 &= \frac{1}{2}(2\tilde{m}_0^2 - X_2 - X_3 + R), \\ m_{\tau_1}^2 &= \frac{1}{2}(2\tilde{m}_0^2 - X_2 - X_3 - R), \\ m_{\tau_2}^2 &= \frac{1}{2}(2\tilde{m}_0^2 + X_2 + X_3 + R), \end{aligned} \quad (3.80)$$

with  $R = \sqrt{4A_y^2 + (X_2 - X_3)^2}$ .

We have the expressions for the s-lepton particle masses and their eigenstates. We are now up to change the lagrangians of interaction with this new information given from the ansatz.

### 3.6.1 Modified Lagrangians

It is of our interest the interactions between the higgs boson and the superparticles because those interactions will be used in our one-loop correction calculation. Furthermore we are interested in the interaction of the Bino particle, the leptons and the superparticles. Therefore we use the *Ansatz* given in section

3.6 and extend the Lagrangians.

The Supersymmetric Lagrangian which models the interaction of the Higgs boson with the  $\tilde{\mu}_j, \tilde{\tau}_j$ , where  $j = 1, 2$  is given by [22]

$$\begin{aligned} \mathcal{L}_{h^0 \tilde{f} \tilde{f}} &= [Q_\mu + G(-\frac{1}{2} + s_w^2)]\tilde{\mu}_L^* \tilde{\mu}_L h^0 + [Q_\mu - G s_w^2]\tilde{\mu}_R^* \tilde{\mu}_R h^0 - H_\mu[\tilde{\mu}_L^* \tilde{\mu}_R h^0 + \tilde{\mu}_R^* \tilde{\mu}_L h^0] \\ &+ [Q_\tau + G(-\frac{1}{2} + s_w^2)]\tilde{\tau}_L^* \tilde{\tau}_L h^0 + [Q_\tau - G s_w^2]\tilde{\tau}_R^* \tilde{\tau}_R h^0 - H_\tau[\tilde{\tau}_L^* \tilde{\tau}_R h^0 + \tilde{\tau}_R^* \tilde{\tau}_L h^0] \end{aligned} \quad (3.81)$$

where

$$Q_{\mu,\tau} = \frac{g m_{\mu,\tau}^2 \sin \alpha}{M_w \cos \beta} \quad (3.82)$$

$$G = g_z M_z \sin(\alpha + \beta) \quad (3.83)$$

$$H_{\mu,\tau} = \frac{g m_{\mu,\tau}}{2 M_w \cos \beta} (A_{\mu,\tau} \sin \alpha - \mu_{susy} \cos \alpha) \quad (3.84)$$

$$(3.85)$$

Using the couplings that we obtained in Matrix 3.78, substituing  $\tilde{\mu}_R, \tilde{\mu}_L, \tilde{\tau}_R$  and  $\tilde{\tau}_L$  and after some algebra we obtain

$$\begin{aligned} \mathcal{L}_{h^0 \tilde{f} \tilde{f}} &= \{s_\varphi^2(Q_\tau + H_\tau) + c_\varphi^2(Q_\mu + H_\mu) - \frac{1}{4}G\}h^0 \tilde{\mu}_1 \tilde{\mu}_1 \\ &+ \{s_\varphi^2(Q_\tau - H_\tau) + c_\varphi^2(Q_\mu - H_\mu) - \frac{1}{4}G\}h^0 \tilde{\mu}_2 \tilde{\mu}_2 \\ &+ \{s_\varphi^2(Q_\mu - H_\mu) + c_\varphi^2(Q_\tau - H_\tau) - \frac{1}{4}G\}h^0 \tilde{\tau}_1 \tilde{\tau}_1 \\ &+ \{s_\varphi^2(Q_\mu + H_\mu) + c_\varphi^2(Q_\tau + H_\tau) - \frac{1}{4}G\}h^0 \tilde{\tau}_2 \tilde{\tau}_2 \\ &+ \frac{1}{4}G(1 - 4s_w^2)h^0 \tilde{\mu}_1 \tilde{\mu}_2 \\ &+ c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\tau - H_\mu)h^0 \tilde{\mu}_1 \tilde{\tau}_2 \\ &+ \frac{1}{4}G(1 - 4s_w^2)h^0 \tilde{\mu}_2 \tilde{\mu}_1 \\ &+ c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\tau - H_\mu)h^0 \tilde{\mu}_2 \tilde{\tau}_1 \\ &+ c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\tau - H_\mu)h^0 \tilde{\tau}_1 \tilde{\mu}_2 \\ &+ \frac{1}{4}G(1 - 4s_w^2)h^0 \tilde{\tau}_1 \tilde{\tau}_2 \\ &+ c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\tau - H_\mu)h^0 \tilde{\tau}_2 \tilde{\mu}_1 \\ &+ \frac{1}{4}G(1 - 4s_w^2)h^0 \tilde{\tau}_2 \tilde{\tau}_1 \end{aligned} \quad (3.86)$$

The Lagrangian that modelates the interaction of  $\tilde{B} \tilde{f} \tilde{f}$  is , where  $\tilde{f} = \tilde{\mu}, \tilde{\tau}$

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \tilde{B}^0 \left\{ [-\tan \theta_w P_L] \tilde{\mu}_L^* \mu + [2 \tan \theta_w P_R] \tilde{\mu}_R^* \mu + [-\tan \theta_w P_L] \tilde{\tau}_L^* \tau + [2 \tan \theta_w P_R] \tilde{\tau}_R^* \tau \right\} \quad (3.87)$$



$g_{h^0 \tilde{f} \tilde{f}}$	$\tilde{\mu}_1$	$\tilde{\mu}_2$	$\tilde{\tau}_1$	$\tilde{\tau}_2$
$\tilde{\mu}_1$	$s_\varphi^2(Q_\tau + H_\tau) + c_\varphi^2(Q_\mu + H_\mu) - \frac{1}{4}G$	$\frac{1}{4}G(1 - 4s_w^2)$	0	$c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\tau - H_\mu)$
$\tilde{\mu}_2$	$\frac{1}{4}G(1 - 4s_w^2)$	$s_\varphi^2(Q_\tau - H_\tau) + c_\varphi^2(Q_\mu - H_\mu) - \frac{1}{4}G$	$c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\mu - H_\tau)$	0
$\tilde{\tau}_1$	0	$c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\mu - H_\tau)$	$s_\varphi^2(Q_\mu - H_\mu) + c_\varphi^2(Q_\tau - H_\tau) - \frac{1}{4}G$	$\frac{1}{4}G(1 - 4s_w^2)$
$\tilde{\tau}_2$	$c_\varphi s_\varphi(Q_\tau - Q_\mu + H_\tau - H_\mu)$	0	$\frac{1}{4}G(1 - 4s_w^2)$	$s_\varphi^2(Q_\mu + H_\mu) + c_\varphi^2(Q_\tau + H_\tau) - \frac{1}{4}G$

Table 3.2: Expressions of the respective interactions of the higgs boson  $h^0$  with the s-fermions

Substitung  $\tilde{\mu}_R, \tilde{\mu}_L, \tilde{\tau}_R$  and  $\tilde{\tau}_L$ , which can be found from Matrix 3.78, and after some algebraic steps we obtain that the lagrangian is

$$\mathcal{L}_{\tilde{B}\tilde{f}\tilde{f}} = - \frac{g}{4} \tilde{B} \tan\theta_w \left\{ c_\varphi(3 + \gamma_5)\tilde{\mu}_1\mu + s_\varphi(3 + \gamma_5)\tilde{\mu}_1 + c_\varphi(1 + 3\gamma_5)\tilde{\mu}_2\mu + s_\varphi(1 + 3\gamma_5)\tilde{\mu}_2\tau \right. \\ \left. - s_\varphi(1 + 3\gamma_5)\tilde{\tau}_1\mu + c_\varphi(1 + 3\gamma_5)\tilde{\tau}_1\tau - s_\varphi(3 + \gamma_5)\tilde{\tau}_2\mu + c_\varphi(3 + \gamma_5)\tilde{\tau}_2\tau \right\} \quad (3.88)$$

$g_{\tilde{B}\tilde{f}\tilde{f}}$	$\mu$	$\tau$
$\tilde{\mu}_1$	$-\frac{g c_\varphi}{4} \tan\theta_w [3 + \gamma_5]$	$-\frac{g s_\varphi}{4} \tan\theta_w [3 + \gamma_5]$
$\tilde{\mu}_2$	$-\frac{g c_\varphi}{4} \tan\theta_w [1 + 3\gamma_5]$	$-\frac{g s_\varphi}{4} \tan\theta_w [1 + 3\gamma_5]$
$\tilde{\tau}_1$	$\frac{g s_\varphi}{4} \tan\theta_w [1 + 3\gamma_5]$	$-\frac{g c_\varphi}{4} \tan\theta_w [1 + 3\gamma_5]$
$\tilde{\tau}_2$	$\frac{g s_\varphi}{4} \tan\theta_w [3 + \gamma_5]$	$-\frac{g c_\varphi}{4} \tan\theta_w [3 + \gamma_5]$

Table 3.3: Expressions obtained from the Lagrangian, in order to use in the different interactions of the  $\tilde{B}$

