

Appendix B

One-Loop Integrals using LoopTools

In Chapter 4 we made the generalized calculation of the contributions to the branching ratio of $h^0 \rightarrow \tau\mu$. However, as it can be noticed many of the expressions are in terms of functions so called $B0$ and $C0$. These multivariable functions are the so called *Passarino-Veltman functions*.

We used the package called *LoopTools* to calculate these integrals. This package was created by Dr. Thomas Hahn. [39] and it calculates numerically this integrations.

In general the subtraction in terms function $B0$ is given by

$$\begin{aligned}
 B0(m_i^2, m_1^2, m_2^2) - B0(m_j^2, m_1^2, m_2^2) &= (m_1^2 - m_2^2) \frac{m_i^2 - m_j^2}{m_i^2 m_j^2} \ln\left[\frac{m_1}{m_i}\right] \\
 &+ \frac{\Lambda_j}{m_j^2} \{ \ln[2m_1 m_2] - \ln[m_1^2 + m_2^2 - m_j^2 + \Lambda_j] \} \\
 &- \frac{\Lambda_i}{m_i^2} \{ \ln[2m_1 m_2] - \ln[m_1^2 + m_2^2 - m_i^2 + \Lambda_i] \}
 \end{aligned} \tag{B.1}$$

where $\Lambda_{i,j} = \sqrt{[m_{i,j}^2 - (m_1^2 + m_2^2)]^2 - 4m_1^2 m_2^2}$

The $C0$ function is defined as

$$C0(p_1, p_2, m_1, m_2, m_3) = \int \frac{d^n q}{(q^2 + m_1 - i\epsilon)((q + p_1)^2 + m_2^2 - i\epsilon)((q + p_1 + p_2)^2 + m_3^2 - i\epsilon)} \tag{B.2}$$

where p_1, p_2, p_3 are the momentums of the loop, m_1, m_2, m_3 the masses of the loop particles. The ϵ variable is an infinitesimal value. In our case it is despreciable and we take it as zero. The solution to this integral is given by

$$C0(p_1, p_2, m_1, m_2, m_3) = \frac{i\pi^2}{2} [\mathcal{L}(y_{01}, y_{11}, y_{21}) - \mathcal{L}_3(y_{02}, y_{12}, y_{22}) - \mathcal{L}(y_{03}, y_{13}, y_{23})] \tag{B.3}$$

where

$$\begin{aligned}
y_{01} &= -\frac{\alpha(-2p_1 \cdot p_2) - 2p_2^2 + (m_2 - m_3^2 + p_2^2 + \alpha(m_1^2 - m_2^2 + p_1^2 + 2p_1p_2))}{-2p_1 \cdot p_2 - 2\alpha p_1^2} \\
y_{02} &= -\frac{m_2^2 - m_3^2 + p_2^2 + \alpha(m_1^2 - m_2^2 + p_1^2 + 2p_1p_2)}{\alpha(-2 \cdot p_2) - 2p_2^2} \\
y_{03} &= -\frac{d - 2\alpha p_1 \cdot p_2}{-2p_1 \cdot p_2 - 2\alpha p_1^2 - 2\alpha p_1 \cdot p_2 - 2p_2^2} \\
y_{11} &= \frac{m_1^2 - m_2^2 + p_1^2 + 2p_1p_2 - 2p_1 \cdot p_2 - \sqrt{(m_1^2 - m_2^2 + 2p_1^2 + 2p_1p_2 + p_2^2 - (p_1 + p_2)^2)^2 + 4p_1^2m_2^2}}{2p_1^2} \\
y_{21} &= \frac{m_1^2 - m_2^2 + p_1^2 + 2p_1p_2 - 2p_1 \cdot p_2 + \sqrt{(m_1^2 - m_2^2 + 2p_1^2 + 2p_1p_2 + p_2^2 - (p_1 + p_2)^2)^2 + 4p_1^2m_2^2}}{2p_1^2} \\
y_{12} &= \frac{m_2^2 - m_3^2 + p_2^2 - \sqrt{(m_2^2 - m_3^2 + p_2^2)^2 + 4p_2^2m_3^2}}{2p_2^2} \\
y_{22} &= \frac{m_2^2 - m_3^2 + p_2^2 + \sqrt{(m_2^2 - m_3^2 + p_2^2)^2 + 4p_2^2m_3^2}}{2p_2^2} \\
y_{13} &= \frac{m_3^2 + m_1^2 + (p_1 + p_2)^2 - \sqrt{(m_3^2 + m_1^2 + (p_1 + p_2)^2)^2 + 4(p_1 + p_2)^4m_3^2}}{2(p_1 + p_2)^2} \\
y_{23} &= \frac{m_3^2 + m_1^2 + (p_1 + p_2)^2 + \sqrt{(m_3^2 + m_1^2 + (p_1 + p_2)^2)^2 + 4(p_1 + p_2)^4m_3^2}}{2(p_1 + p_2)^2}
\end{aligned} \tag{B.4}$$

and \mathcal{L} is given by

$$\mathcal{L} = \int_0^1 dy \frac{1}{y - y_0} [\ln(ay^2 + by + c) - \ln(ay_0^2 + by_0 + c)] \tag{B.5}$$

where we need to equal the expression to obtain the coefficients a, b, c

$$ay^2 + by + c = a(y - y_1)(y - y_2) \tag{B.6}$$