Appendix A

Explicit Calculation of the Amplitudes of $h^0 \rightarrow \tau \mu$

Our interest in this appendix is to show the explicit calculation of the different amplitudes of the sixteen possible Feynmann Diagrams with Flavour Violation in MSSM taking our Ansatz [1]. The calculation will lead us to a constant which we call as $\alpha_{jk}$, the scalar $S_{jk}$ and pseudoscalar $P'_{jk}$ part of the amplitudes. After we know the scalar and pseudoscalar part of each possible Feynman Diagram we proceed to calculate

$$\Gamma(h^0 \rightarrow \mu\tau) = \frac{|\alpha_{jk}|^2 \rho}{8\pi^2 m_{h^0}^2} \left\{ (|S_{jk}|^2 + |P'_{jk}|^2) (E_{\tau} E_{\mu} + \rho^2) + (|P'_{jk}|^2 - |S_{jk}|^2) m_{\tau} m_{\mu} \right\} \tag{A.1}$$

where

$$\rho = \sqrt{\frac{[m_{h^0}^2 - (m_{\mu} + m_{\tau})^2][m_{h^0}^2 - (m_{\mu} - m_{\tau})^2]}{2m_{h^0}^2}}$$

$$m_{\tau} = 1.77699 \frac{GeV}{c^2}$$

$$m_{\mu} = 0.1056583715 \frac{GeV}{c^2}$$

$$m_{h^0} = 125 \frac{GeV}{c^2} \tag{A.2}$$
The amplitude is the integration over the momentums of the quantum loop correction.

\[ M_{11} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}(k_1) \cdot g_{\tilde{B}\tilde{B}_1} \cdot \bar{P}_{\tilde{B}}(q_2) \cdot g_{\tilde{B}\tilde{B}_1\tau} \cdot \bar{u}_\tau(k_2) \cdot P_{\tilde{B}_1}(q_3) \cdot g_{h^0\tilde{B}_1\tilde{B}_1} \cdot P_{\tilde{B}_1} \]

(A.3)

where \( g_{\tilde{B}\tilde{B}_1\rho} \), \( g_{\tilde{B}\tilde{B}_1\tau} \), \( g_{h^0\tilde{B}_1\tilde{B}_1} \) represent the interactions and are taken from the respective Lagrangians. \( P_{\tilde{B}}, P_{\tilde{B}_1} \) represent the propagators. In table 4.3 are shown the propagators that are of our interest.

We substitute the respective propagators, taken from table 4.3:

\[ M_{11} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}(k_1) \cdot g_{\tilde{B}\tilde{B}_1} \cdot \frac{i(q^2 + m_B^2)}{q^2 - m_B^2} \cdot g_{\tilde{B}\tilde{B}_1\tau} \cdot \bar{u}_\tau(k_2) \cdot \frac{i}{q_1^2 - m_{\tilde{B}_1}^2} \cdot g_{h^0\tilde{B}_1\tilde{B}_1} \cdot \frac{i}{q_3^2 - m_{\tilde{B}_1}^2} \]

(A.4)

From \( \mathcal{L}_{\tilde{B}\tilde{f}_f} \) we have that the interaction \( g_{\tilde{B}\tilde{B}_1} \) is

\[ g_{\tilde{B}\tilde{B}_1} = -\frac{gc_2}{4}\tan\theta_w[3 + \gamma_5] \]

(A.5)

Similarly from \( \mathcal{L}_{\tilde{B}\tilde{f}_f} \), we obtain the following expression

\[ g_{\tilde{B}\tilde{B}_1\tau} = -\frac{gs_2}{4}\tan\theta_w[3 + \gamma_5] \]

(A.6)

Substituting (A.5), (A.6) in (A.3) we obtain:

\[ M_{11} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}(k_1) \cdot \frac{gc_2}{4} \tan\theta_w(3 + \gamma_5) \cdot \frac{i(q^2 + m_B^2)}{q^2 - m_B^2} \cdot \frac{gs_2}{4} \tan\theta_w(3 + \gamma_5) \cdot \bar{u}_\tau(k_2) \cdot \frac{1}{q_1^2 - m_{\tilde{B}_1}^2} \cdot \frac{1}{q_3^2 - m_{\tilde{B}_1}^2} \]

(A.7)
Labeling the following expressions as $\alpha_{11}, N_1$ and $D_1$:

$$\alpha_{11} = -g_{\mu\nu}\bar{\mu}_1 \mu \frac{i\gamma^2 s_{\nu} c_{\nu}}{16} \tan^2 \theta$$

$$N_{11} = \bar{\nu}_\mu(k_1)(3 + \gamma_5)(q^2 + m_B)(3 + \gamma_5)u_\tau(k_2)$$

$$D_{11} = (q^2 - m_B^2)(q^2 - m_{\mu_1}^2)(q^2 - m_{\nu_1}^2) \tag{A.8}$$

where $g_{\mu\nu}\bar{\mu}_1 \mu = s_{\nu}^2 (Q_\tau + X_\tau) + c_{\nu}^2 (Q_\mu + X_\mu) - \frac{1}{4}G$

We can express the amplitude as:

$$M_{11} = \alpha_{11} \int \frac{d^4 q_1}{(2\pi)^4} \frac{N_{11}}{D_{11}} \tag{A.9}$$

Expanding the products of $N_1$ and substituting $q_3 = k_1 + k_2 + q_1$, $q_2 = k_2 + q_1$

$$N_{11} = \bar{\nu}_\mu(k_1)(3 + \gamma_5)(q^2 + m_B)(3 + \gamma_5)u_\tau(k_2)$$

$$\quad = \bar{\nu}_\mu(k_1) \{9(q^2 + k^2 + m_B^2) + 3(q^2 + k^2 + m_B^2)\gamma_5 + \gamma_5(q^2 + k^2 + m_B^2)3$$

$$\quad + \gamma_5(q^2 + k^2 + m_B^2)\gamma_5\} u_\tau(k_2)$$

We use two properties of the Dirac matrices

$$\gamma^2 = 1$$

$$\gamma_5\gamma^\mu = -\gamma^\mu\gamma_5 \tag{A.10}$$

And substituting

$$\gamma^\mu q_1\mu, \gamma^\mu k^\mu_2$$

$$N_{11} = \bar{\nu}_\mu(k_1) \{9(\gamma^\mu q_1\mu + \gamma^\mu k^\mu_2 + m_B^2) + 3(\gamma^\mu q_1\mu + \gamma^\mu k^\mu_2 + m_B^2)\gamma_5 + \gamma_5(\gamma^\mu q_1\mu + \gamma^\mu k^\mu_2 + m_B^2)3$$

$$\quad + \gamma_5(\gamma^\mu q_1\mu + \gamma^\mu k^\mu_2 + m_B^2)\gamma_5\} u_\tau(k_2)$$

$$\quad = \bar{\nu}_\mu(k_1) \{8\gamma^\mu q_1\mu + 8\gamma^\mu k^\mu_2 + 10m_B + 6\gamma_5 m_B\} u_\tau(k_2)$$

$$\quad = \bar{\nu}_\mu(k_1) \{8q^2 + 8k^2 + 10m_B + 6\gamma_5 m_B\} u_\tau(k_2) \tag{A.11}$$

If we substitute the expression above in $M_{11}$ and $q_3 = k_1 + k_2 + q_1$, $q_2 = k_2 + q_1$ in $D_{11}$

$$\alpha_{11} \int \frac{\bar{\nu}_\mu(k_1) \{8q^2 + 8k^2 + 10m_B + 6\gamma_5 m_B\} u_\tau(k_2)}{((k_2 + q_1)^2 - m_B^2)(q^2 - m_{\mu_1}^2)((k_2 + k_1 + q_1)^2 - m_{\nu_1}^2)} \tag{A.12}$$
We separate the integral, since the integral is a linear operator.

\[ M_{11} = \alpha_1 \bar{v}_\mu(k_1) \left\{ 8 \int dq_1 \frac{q_1^4}{(2\pi)^4 ((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_{\mu_1}^2)((q_1 + k_2 + k_1)^2 - m_{\mu_1}^2)} \right\} u_\tau(k_2) \]

We use the completeness relation that states that \( \hat{k}_\mu u_\tau(k_2) = m_\tau u_\tau(k_2) \)

\[ M_{11} = -8i\pi^2 \frac{c_{h_\mu_\tau}}{C_{h_\mu_\tau}} \left\{ B_{11} - F_{\sigma_0} [C_{11} + C_{h_\mu_\tau} (m_\tau + \frac{10}{8} m_B)] \right\} + 6i\pi^2 F_{\sigma_0} \gamma_5 m_B \]

We use the completeness relation that states that \( \hat{k}_\mu u_\tau(k_2) = m_\tau u_\tau(k_2) \)

\[ M_{11} = -8c_{h_{\mu_{\tau}}} \left\{ B_{11} - F_{\sigma_0} [C_{11} + C_{h_{\mu_{\tau}}} (m_\tau + \frac{10}{8} m_B)] \right\} + 6i\pi^2 F_{\sigma_0} \gamma_5 m_B \]

where we called as \( S_{11} = -8c_{h_{\mu_{\tau}}} \left\{ B_{11} - F_{\sigma_0} [C_{11} + C_{h_{\mu_{\tau}}} (m_\tau + \frac{10}{8} m_B)] \right\} \) and \( P_{11} = 6i\pi^2 F_{\sigma_0} \gamma_5 m_B \) because one is scalar and the second one is pseudoscalar.
We continue with amplitude $M_{12}$. We calculate it with the following equation.

$$M_{12} = \int \frac{d^4q_1}{(2\pi)^4} \times \bar{v}_\mu(k_1) \times g_{B\bar{\rho}_1\mu} \times P^\_B(q_2) \times g_{B\bar{\rho}_2\tau} \times u_\tau(k_2) \times P^\_\bar{\mu}_1(q_1) \times g_{h\epsilon\bar{\nu}_1\bar{\nu}_2} \times P^\_\bar{\mu}_3(q_3)$$

(A.16)

From Table 3.3 we have that

$$g_{B\bar{\rho}_1\mu} = -\frac{g c_\phi}{4} \tan \theta_w [3 + \gamma_5]$$

(A.17)

$$g_{B\bar{\rho}_2\tau} = -\frac{g s_\phi}{4} \tan \theta_w [1 + 3\gamma_5]$$

(A.18)

And from Table 4.3 we obtain

$$P^\_B(q_2) = \frac{i(q_2^2 + m_B^2)}{q_2^2 - m_B^2}$$

(A.20)

$$P^\_\bar{\mu}_1(q_1) = \frac{i}{q_1^2 - m_{\bar{\mu}_1}^2}$$

(A.21)

$$P^\_\bar{\mu}_3(q_3) = \frac{i}{q_3^2 - m_{\bar{\mu}_3}^2}$$

(A.22)

Substituting the propagators and two of the three interactions in the integral, we obtain

$$M_{12} = -\int \frac{d^4q_1}{(2\pi)^4} \times \bar{v}_\mu(k_1) \times g_{h\epsilon\bar{\nu}_1\bar{\nu}_2} \times \frac{g c_\phi}{16} \tan \theta_w [3 + \gamma_5] \times \frac{i(q_2^2 + m_B^2)}{q_2^2 - m_B^2} \times$$

$$\times \frac{g s_\phi}{4} \tan \theta_w [1 + 3\gamma_5] \times u_\tau(k_2) \times \frac{i}{q_1^2 - m_{\bar{\mu}_1}^2} \times \frac{i}{q_3^2 - m_{\bar{\mu}_3}^2} \times$$

$$\times [1 + 3\gamma_5] \times u_\tau(k_2) \times \frac{1}{q_1^2 - m_{\bar{\mu}_2}^2} \times \frac{1}{q_3^2 - m_{\bar{\mu}_1}^2}$$

(A.23)
We have that:

\[ M_{12} = \alpha_{12} \int \frac{d^4q_1}{(2\pi)^4} D_{12} \]

(A.25)

Working with \( N_{12} \) and leaving it in terms of \( q_1 \), using (A.6)

\[ N_{12} = \bar{v}_\mu(k_1)[3 + \gamma_5][\gamma_2 + m_B][1 + 3\gamma_5]u_\tau(k_2) \]

(A.26)

We substitute \( N_{12} \) in \( M_{12} \) and we separate the integral

\[ M_{12} = \alpha_{12} \bar{v}_\mu(k_1) \left\{ \begin{array}{l}
-8\gamma_5\gamma_2 \int \frac{d^4q_1}{(2\pi)^4} \frac{1}{[(k_2 + q_1)^2 - m_B^2][\gamma^2 - m_{\mu_2}^2][(k_2 + k_1 + q_1)^2 - m_{\mu_1}^2]}
\end{array} \right. 
\]

(A.28)

Using the result of the generalized integrals in Eq. (A.26) (A.38) we obtain

\[ M_{12} = \alpha_{12} \bar{v}_\mu(k_1) \left\{ \begin{array}{l}
-8i\pi^2 F_{\alpha\gamma_5} \gamma_2 + 8i\pi^2 C_{H^0\gamma_5} \gamma_5 \{B_{12} - F_{\rho\alpha} C_{12}\} \gamma_5 \\
+ 10i\pi^2 m_B F_{\alpha\gamma_5} + 6m_B \pi^2 F_{\alpha_0} \\
+ 10i\pi^2 m_B F_{\alpha_0\gamma_5} + 6m_B \pi^2 F_{\alpha_0} \gamma_5 \\
= \alpha_{12} \bar{v}_\mu(k_1) \left\{ \begin{array}{l}
6i\pi^2 m_B F_{\alpha_0} + 8i\pi^2 C_{H^0\gamma_5} \gamma_5 \{B_{12} - F_{\rho\alpha_0} C_{12} + C_{H^0\mu_\tau}(m_\tau - 10m_B)\} \gamma_5 \end{array} \right. 
\right. 
\]

(A.29)
where we used the completeness relation \( \vec{k}_2 \gamma \cdot \vec{u}_{\tau}(k_2) = m_{\tau} \gamma \cdot \vec{u}_{\tau}(k_2) \) Or separating in the scalar part and pseudoscalar part

\[
S_{12} = 6 i m_\beta \pi^2 F_{c0} \\
P_{12} = 8 \frac{i \pi^2}{C_{h^0 \mu \tau}} \left( B_{12} - F_{c0} (C_{12} + C_{h^0 \mu \tau} (m_{\tau} - \frac{10}{8} m_\beta)) \right) \gamma_5
\]

(A.30)

\[
M_{12} = a_{12} \bar{v}_{\mu}(k_1) \{ S_{12} + P_{12} \} u_{\tau}(k_2)
\]

(A.31)
We have that
\[ M_{13} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\nu}_\mu(k_1) * g_{\tilde{B}\tilde{\tau}_1} \tau_1 * u_\tau(k_2) * P_\tau(q_1) * g_{h^0\tilde{\mu}_1} \tilde{\mu} (q_3) \] (A.32)

However, from table 3.2 we have that the interaction of higgs \( h^0, \tilde{\mu}_1, \tilde{\tau}_1 \) is zero, since it does not exist in the Lagrangian. Therefore \( g_{h^0\tilde{\mu}_1} \tilde{\tau}_1 = 0 \) and we obtain
\[ M_{13} = 0 \] (A.33)

For simplicity it will be written
\[ \alpha_{13} = 0 \]
\[ S_{13} = 0 \]
\[ P_{13} = 0 \] (A.34)
We have that the amplitude $M_{14}$ is

$$M_{14} = \int \frac{d^4q_1}{(2\pi)^4} \times \bar{\psi}_\mu(k_1) \times g_{\bar{B}\bar{\mu}1} \times P_{\bar{B}}(q_2) \times g_{B\bar{\tau}2} \times u_\tau(k_2) \times P_\tau(k_1) \times g_{B\bar{\mu}1} \times P_{\bar{B}}(q_3)$$

(A.35)

From Table 3.3, we have that

$$g_{\bar{B}\bar{\mu}1} = \frac{gc_\varphi}{4} \tan\theta_w \bar{\psi} \mu(k_1) \times 3 + \gamma_5$$

(A.36)

$$g_{B\bar{\tau}2} = \frac{gc_\varphi}{4} \tan\theta_w \bar{\psi} \tau(k_1) \times 3 + \gamma_5$$

(A.37)

From Table 4.3, we obtain

$$P_{\bar{B}}(q_2) = \frac{q_2^2 - m^2_{\bar{B}}}{q_2^2 - m^2_B}$$

(A.38)

$$P_\tau(k_1) = \frac{i}{q_1^2 - m^2_\tau}$$

(A.39)

$$P_{\bar{\mu}_1}(k_3) = \frac{i}{q_3^2 - m^2_{\bar{\mu}_1}}$$

(A.40)

From Table 3.2,

$$g_{B\bar{\mu}_1\tau} = c_\varphi s_\varphi (Q_\bar{\tau} - Q_\bar{\mu} + X_\bar{\tau} - X_\bar{\mu})$$

(A.41)

Substituting the propagators and the respective interactions, we obtain

$$M_{14} = \int \frac{d^4q_1}{(2\pi)^4} \psi_\mu(k_1) \frac{gc_\varphi}{4} \tan\theta_w [3 + \gamma_5] \frac{i(q_2^2 + m^2_B)}{q_2^2 - m^2_B} \frac{gc_\varphi}{4} \tan\theta_w [3 + \gamma_5] u_\tau(k_2) \times$$

$$\times \frac{i}{q_1^2 - m^2_\tau}$$

$$= \int \frac{d^4q_1}{(2\pi)^4} i g_{B\bar{\mu}_1\tau} g^2 c^2 \bar{\psi}_\mu(k_1) [3 + \gamma_5] \frac{q_2^2 + m^2_B}{q_2^2 - m^2_B} [3 + \gamma_5] \times$$

$$\times u_\tau(k_2) \frac{1}{q_1^2 - m^2_\tau} \frac{q_3^2}{q_3^2 - m^2_{\bar{\mu}_1}}$$

(A.42)
Labeling the following expressions as $\alpha_{14}, N_{14}$ and $D_{14}$

$$N_{14} = \bar{v}_{\mu}(k_1)[3 + \gamma_5][g_2 + m_B][3 + \gamma_5]u_{\tau}(k_2)$$

$$\alpha_{14} = -\frac{ig_{\nu_{\mu}+g_2c_2^2}}{16}\tan^2\theta_w$$

$$D_{14} = [q_2^2 - m_B^2][q_1^2 - m_B^2][q_3^2 - m_{\mu^2}]$$  \hspace{1cm} (A.43)

Therefore, we can rewrite $M_{14}$ as

$$M_{14} = \alpha_{14} \int \frac{d^4q_1}{(2\pi)^4} \frac{N_{14}}{D_{14}}$$  \hspace{1cm} (A.44)

As it can be noticed $N_{14}$ is exactly as $N_{11}$ in [A.8]. Therefore substituing $N_{11}$ in the expression above

$$M_{14} = \alpha_{14}\bar{v}_{\mu}(k_1)\left\{8\int \frac{d^4q_1}{(2\pi)^4} \frac{q_1^2}{((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_B^2)((q_1 + k_2 + k_1)^2 - m_{\mu^2})} + 8k_2^2 \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_1}{((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_B^2)((q_1 + k_2 + k_1)^2 - m_{\mu^2})} + 10m_B \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_1}{((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_B^2)((q_1 + k_2 + k_1)^2 - m_{\mu^2})} + 6\gamma_5m_B \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_1}{((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_B^2)((q_1 + k_2 + k_1)^2 - m_{\mu^2})}}{u_{\tau}(k_2)} \right\}$$  \hspace{1cm} (A.45)

Using the integral generalization in Eq. 4.26 and Eq. 4.38 for this particular case ($j=1,k=4$)

$$M_{14} = \alpha_{14}\bar{v}_{\mu}(k_1)\left\{ -8\frac{i\pi^2}{C_{h^0_{\mu\tau}}} \left\{ B_{14} - F_{C0}C_{14} \right\} + i\pi^2F_{C0}(8k_2 + 10m_B + 6\gamma_5m_B) \right\}u_{\tau}(k_2)$$  \hspace{1cm} (A.46)

We use the completeness relation that states that $\hat{k}_2u_{\tau}(k_2) = m_{\tau}u_{\tau}(k_2)$

$$M_{14} = \alpha_{14}\bar{v}_{\mu}(k_1)\left\{ -8\frac{i\pi^2}{C_{h^0_{\mu\tau}}} \left\{ B_{14} - F_{C0}[C_{14} + C_{h^0_{\mu\tau}}(m_{\tau} + \frac{10}{8}m_B)] \right\} + 6i\pi^2F_{C0}\gamma_5 \right\}u_{\tau}(k_2)$$

$$= \alpha_{14}\bar{v}_{\mu}(k_1)\left\{ S_{14} + P_{14} \right\}u_{\tau}(k_2)$$  \hspace{1cm} (A.47)

where we separeted the scalar and pseudoscalar part of $M_{14}$ in $S_{14}$ and $P_{14}$.
We follow the same algorithm for calculating the amplitude. The amplitude is represented by $M_{21}$ and is calculated as follows

$$M_{14} = \int \frac{d^4q_1}{(2\pi)^4} \times \bar{v}_\mu(k_1) \times g_{\tilde{B}{\tilde{B}}_2\mu} \times P_\tilde{B}(q_2) \times g_{\tilde{B}{\tilde{B}}_1\tau} \times u_\tau(k_2) \times P_{\tilde{B}_1}(q_1) \times g_{h{\tilde{B}_1}{\tilde{B}_2}} \times P_{\tilde{B}_2}(q_3)$$

(A.48)

We take the propagators and interactions from tables 4.3, 3.3.

$$g_{\tilde{B}{\tilde{B}}_2\mu} = -\frac{gc_\varphi}{4} \tan\theta_w [1 + 3\gamma_5]$$

(A.49)

$$g_{\tilde{B}{\tilde{B}}_1\tau} = -\frac{gs_\varphi}{4} \tan\theta_w [3 + \gamma_5]$$

(A.50)

$$P_B(q_2) = \frac{i(q_2 + m_B)}{q_2^2 - m_B^2}$$

(A.51)

$$P_{\tilde{B}_1}(q_1) = \frac{i}{q_1^2 - m_{\tilde{B}_1}}$$

(A.52)

$$P_{\tilde{B}_2}(q_3) = \frac{i}{q_3^2 - m_{\tilde{B}_2}}$$

(A.53)

Therefore we have that

$$M_{21} = \int \frac{d^4q_1}{(2\pi)^4} \bar{v}_\mu(k_1) \frac{gc_\varphi}{4} \tan\theta_w [1 + 3\gamma_5] \frac{i(q_2 + m_B)}{q_2^2 - m_B^2} \frac{gs_\varphi}{4} \tan\theta_w [3 + \gamma_5] u_\tau(k_2) \times$$

$$\times \frac{i}{q_1^2 - m_{\tilde{B}_1}} \frac{i}{q_3^2 - m_{\tilde{B}_2}}$$

$$= -\int \frac{d^4q_1}{(2\pi)^4} \frac{ig_{h{\tilde{B}_1}{\tilde{B}_2}}}{16} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \frac{c_\varphi s_\varphi}{4} \tan^2\theta_w \bar{v}_\mu(k_1)[1 + 3\gamma_5] \gamma_\rho \gamma_\sigma [3 + \gamma_5] u_\tau(k_2) \frac{1}{q_1^2 - m_{\tilde{B}_1}} \frac{1}{q_3^2 - m_{\tilde{B}_2}}$$

(A.54)

And as we did in the last cases, we label an expression $N_{21,\alpha 21}$ and $D_{21}$

$$\alpha_{21} = -\frac{ig_{h{\tilde{B}_1}{\tilde{B}_2}}}{16} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \frac{c_\varphi s_\varphi}{4} \tan^2\theta_w$$

$$N_{21} = \bar{v}_\mu(k_1)[1 + 3\gamma_5] q_2^\rho m_B [3 + \gamma_5] u_\tau(k_2)$$

$$D_{21} = [q_2^2 - m_B^2][q_1^2 - m_{\tilde{B}_1}][q_3^2 - m_{\tilde{B}_2}]$$

(A.55)
Using the expressions above we can express \( M_{21} \) as

\[
M_{21} = \alpha_{21} \int \frac{d^4q_1}{(2\pi)^4} N_{21}
\]

We expand \( N_{21} \). We use the dirac matrix property \( \gamma_5\gamma^\mu = -\gamma^\mu\gamma_5 \). Also we know that \( \not{k}_2 = \gamma^\mu k_{2\mu}, \not{q}_1 = \gamma^\mu q_{1\mu} \)

\[
N_{21} = \bar{\psi}_\mu(k_1) \{ 1 + 3\gamma_5 \} [\not{q}_2 + m_B] [3 + \gamma_5] u_\tau(k_2)
\]

(A.56)

We substitute \( N_{21} \) in \( M_{21} \) and substitute \( q_3 = k_2 + k_1 + q_1, q_2 = k_2 + q_1 \) in \( D_{21} \)

\[
M_{21} = \alpha_{21} \bar{\psi}_\mu(k_1) \left\{ \begin{array}{c}
8\gamma_5 \int \frac{d^4q_1}{(2\pi)^4} \left[ (k_2 + q_1)^2 - m_B^2 \right] [q_1^2 - m_\mu^2] [q_1^2 - m_\mu^2] [k_2 + k_1 + q_1]^2 - m_\mu^2 \\
10\gamma_5 k_2 \int \frac{d^4q_1}{(2\pi)^4} \left[ (k_2 + q_1)^2 - m_B^2 \right] [q_1^2 - m_\mu^2] [q_1^2 - m_\mu^2] [k_2 + k_1 + q_1]^2 - m_\mu^2 \\
6m_B \int \frac{d^4q_1}{(2\pi)^4} \left[ (k_2 + q_1)^2 - m_B^2 \right] [q_1^2 - m_\mu^2] [q_1^2 - m_\mu^2] [k_2 + k_1 + q_1]^2 - m_\mu^2 \\
\end{array} \right\} u_\tau(k_2)
\]

(A.57)

We use Eq. 4.26 and Eq. 4.38 and substitute the value of the integrals above.

For this particular case \( j = 2 \) and \( k = 1 \).

\[
M_{21} = \alpha_{21} \bar{\psi}_\mu(k_1) \left\{ -8\gamma_5 \frac{i\pi^2}{C_{h^\nu\mu\tau}} \left[ B_{21} - F_{0\alpha} C_{21} \right] + 8\gamma_5 m_\tau i\pi^2 F_{0\alpha} + 10\gamma_5 m_B i\pi^2 F_{0\alpha} + 6i\pi^2 m_B F_{0\alpha} \right\}
\]

(A.58)

and where we used the completeness relation \( \not{k}_2 u_\tau(k_2) = m_\tau u_\tau(k_2) \)

If we label the scalar and pseudoscalar part

\[
S_{21} = 6i\pi^2 m_B F_{0\alpha} \quad P_{21} = -8\frac{i\pi^2}{C_{h^\nu\mu\tau}} \left[ B_{21} - F_{0\alpha} C_{21} + C_{h^\nu\mu\tau}(m_\tau + \frac{10}{8} m_B) \right] \gamma_5
\]

(A.59)

(A.60)

\[
M_{21} = \alpha_{21} \bar{\psi}_\mu(k_1) \{ S_{21} + P_{21} \}
\]

(A.61)
We use equation 4.6 since the momentums are labeled in the same way. We have

\[ M_{22} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_\mu(k_1) g\bar{\psi}_\mu \ast P_B(q_2) \ast g\bar{\psi}_2 \ast u_r(k_1) P_{\mu_2}(q_1) \ast g\bar{\psi}_{\mu_2} \ast P_{\mu_2}(q_3) \]  
(A.62)

From the Lagrangian we have

\[ g\bar{\psi}_\mu \bar{\psi}_\mu = -\frac{g}{4} \tan \theta_w [1 + 3 \gamma_5] \quad (A.63) \]
\[ g\bar{\psi}_2 \bar{\psi}_2 = -\frac{g}{4} \tan \theta_w [1 + 3 \gamma_5] \quad (A.64) \]

Substituting the propagators and the interactions

\[
M_{22} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_\mu(k_1) \frac{g}{4} \tan \theta_w [1 + 3 \gamma_5] \times \\
\times \frac{i}{q_2^2 - m_B^2} \frac{g}{4} g_{\mu \rho \sigma \tau} \tan \theta_w [1 + 3 \gamma_5] u_r(k_2) \frac{i}{q_1^2 - m_{\mu_2}^2} g_{\mu \rho \sigma \tau} \frac{i}{q_3^2 - m_{\mu_2}^2} \\
= -\int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_\mu(k_1) \frac{ig}{8} \frac{g^2}{16} \tan \theta_w [1 + 3 \gamma_5] \times \\
\times \frac{q_2^2 + m_B^2}{q_1^2 - m_{\mu_2}^2} [1 + 3 \gamma_5] u_r(k_2) \frac{1}{q_1^2 - m_{\mu_2}^2} \frac{1}{q_3^2 - m_{\mu_2}^2} \\
(A.65)
\]

Labeling the following expressions as \( \alpha_{22}, D_{22} \) and \( N_{22} \). We can rewrite \( M_{22} \) as

\[
M_{22} = \alpha_{22} \int \frac{d^4q_1}{(2\pi)^4} N_{22} \frac{d^4q_2}{(2\pi)^4} \\
\alpha_{22} = -\frac{g}{16} \frac{g^2}{16} \tan^2 \theta_w \]
\[ N_{22} = \bar{\psi}_\mu(k_1)(1 + 3 \gamma_5)(q_2^2 + m_B^2)(1 + 3 \gamma_5) u_r(k_2) \]
\[ D_{22} = (q_2^2 - m_B^2)(q_1^2 - m_{\mu_2}^2)(q_3^2 - m_{\mu_2}^2) \]  
(A.66)
Expanding the products of $N_{22}$. Using the properties of gamma matrixes \(A.10\)

\[
N_{22} = \bar{v}_\mu(k_1)(1 + 3\gamma_5)(\gamma_2 + m_\tilde{B})(1 + 3\gamma_5)u_\tau(k_2)
= \bar{v}_\mu(k_1)((\gamma_2 + m_\tilde{B}) + (\gamma_2 + m_\tilde{B})3\gamma_5 + 3\gamma_5(\gamma_2 + m_\tilde{B})3\gamma_5)u_\tau(k_2)
= \bar{v}_\mu(k_1)((\gamma_2 + m_\tilde{B}) + 3\gamma_5(\gamma_2 + m_\tilde{B}) + 3\gamma_5(\gamma_2 + m_\tilde{B})3\gamma_5)u_\tau(k_2)
= \bar{v}_\mu(k_1)(-8\gamma_2 + 10m_\tilde{B} + 6\gamma_5m_\tilde{B})u_\tau(k_2)
= \bar{v}_\mu(k_1)(-8(\gamma_2 + q_1) + 10m_\tilde{B} + 6\gamma_5m_\tilde{B})u_\tau(k_2)
\]

We substitute $N_{22}$ in $M_{22}$ and $q_2 = k_2 + q_1$, $\varphi_2 = k_2 + k_1 + q_1$.

\[
M_{22} = \alpha_{22}\bar{v}_\mu(k_1)\left\{-8\gamma_2\int\frac{d^4q_1}{((k_2 + q_1)^2 - m_\tilde{B}^2)}(q_1^2 - m_\mu^2)((k_2 + k_1 + q_1)^2 - m_\mu^2)
-8\int\frac{d^4q_1}{((k_2 + q_1)^2 - m_\tilde{B}^2)}(q_1^2 - m_\mu^2)((k_2 + k_1 + q_1)^2 - m_\mu^2)
+10m_\tilde{B}\int\frac{d^4q_1}{((k_2 + q_1)^2 - m_\tilde{B}^2)}(q_1^2 - m_\mu^2)((k_2 + k_1 + q_1)^2 - m_\mu^2)
+6\gamma_5m_\tilde{B}\int\frac{d^4q_1}{((k_2 + q_1)^2 - m_\tilde{B}^2)}(q_1^2 - m_\mu^2)((k_2 + k_1 + q_1)^2 - m_\mu^2)\right\}u_\tau(k_2)
\]

Using the results of the integrals in Eq. 4.38 and Eq. 4.26 for this particular case ($j=2; k=2$)

\[
M_{22} = \alpha_{22}\bar{v}_\mu(k_1)\left\{-8i\pi^2\gamma_2 F_{\varphi_0} + 8\frac{i\pi^2}{C^{\mu_\varphi_{\mu\tau}}}\{B_{22} - F_{\varphi_0}C_{22}\} + 10m_\tilde{B}i\pi^2 F_{\varphi_0} + 6\gamma_5m_\tilde{B}i\pi^2 F_{\varphi_0}\right\}u_\tau(k_2)
\]

We use the completeness relation $\bar{v}_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$

\[
M_{22} = \alpha_{22}\bar{v}_\mu(k_1)\left\{8\frac{i\pi^2}{C^{\mu_\varphi_{\mu\tau}}}\{B_{22} - F_{\varphi_0}[C_{22} + C^{\mu_\varphi_{\mu\tau}}(m_\tau - \frac{10}{8}m_\tilde{B})]\} + 6\gamma_5m_\tilde{B}i\pi^2 F_{\varphi_0}\right\}u_\tau(k_2)
\]

Where the scalar and pseudoscalar parts are

\[
S_{22} = 8\frac{i\pi^2}{C^{\mu_\varphi_{\mu\tau}}}\{B_{22} - F_{\varphi_0}[C_{22} + C^{\mu_\varphi_{\mu\tau}}(m_\tau - \frac{10}{8}m_\tilde{B})]\}
\]

\[
P_{22} = 6\gamma_5m_\tilde{B}i\pi^2 F_{\varphi_0}
\]

And

\[
M_{22} = \alpha_{22}\bar{v}_\mu(k_1)\{S_{22} + P_{22}\}u_\tau(k_2)
\]
We calculate $M_{23}$ as

$$M_{23} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_\mu(k_1) g_{\tilde{B} \tilde{\mu}} * P_{\tilde{B}}(q_2) * g_{\tilde{B} \tilde{\tau}_1} * u_\tau(k_2) P_{\tilde{\tau}_1}(q_1) * g_{\tilde{\mu} \tilde{\tau}_2} * P_{\tilde{\mu}_2}(q_3)$$

(A.74)

Taking the vertexes and the propagators from tables 3.3, 4.3

$$g_{\tilde{B} \tilde{\mu}} = -\frac{g c^2}{4} \tan \theta_w [1 + 3 \gamma_5]$$
$$g_{\tilde{B} \tilde{\tau}_1} = -\frac{g c^2}{4} \tan \theta_w [1 + 3 \gamma_5]$$
$$P_{\tilde{B}}(q_2) = \frac{i(g_2 + m_{\tilde{B}})}{q_2^2 - m_{\tilde{B}}^2}$$
$$P_{\tilde{\tau}_1}(q_1) = \frac{i}{q_1^2 - m_{\tilde{\tau}_1}^2}$$
$$P_{\tilde{\mu}_2}(q_3) = \frac{i}{q_3^2 - m_{\tilde{\mu}_2}^2}$$

(A.75)

By this way we obtain that the Amplitude $M_{23}$ is

$$M_{23} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_\mu(k_1) \frac{g c^2}{4} \frac{\tan \theta_w [1 + 3 \gamma_5]}{16} \frac{i(g_2 + m_{\tilde{B}})}{q_2^2 - m_{\tilde{B}}^2} \frac{g c^2 \tan \theta_w [1 + 3 \gamma_5] u_\tau(k_2)}{q_1^2 - m_{\tilde{\tau}_1}^2} \frac{1}{q_3^2 - m_{\tilde{\mu}_2}^2}$$

(A.76)
APPENDIX A. EXPLICIT CALCULATION OF THE AMPLITUDES OF $H^0 \rightarrow \tau \mu$

Labeling $N_{23}, \alpha_{23}$ and $D_{23}$ as

$$
N_{23} = \bar{v}_\mu(k_1)[1 + 3\gamma_5][g_2 + m_B][1 + 3\gamma_5]u_\tau(k_2)
$$

$$
\alpha_{23} = -\frac{i g_\rho \bar{v}_\mu \gamma_\rho (2\eta - \tau \mu)}{2m_B^2} + \tau \mu + \tau \mu
$$

$$
D_{23} = [q_2^2 - m_B^2][q_1^2 - m_{\bar{\mu} \tau}^2][q_2^2 - m_{\bar{\mu} \tau}^2]
$$

As it can be noticed in Eq. A.66 that $N_{23} = N_{22}$. Therefore substituting $N_{22}$ in $M_{23}$

$$
M_{23} = \alpha_{23} \bar{v}_\mu(k_1) \left\{ -8k_2 \int (k_2 + q_1)^2 - m_B^2) (q_1^2 - m_B^2) \right\} u_\tau(k_2)
$$

Using the results of the integrals in Eq. 4.38 and Eq. 4.26 for this particular case ($j=2, k=3$)

$$
M_{23} = \alpha_{23} \bar{v}_\mu(k_1) \left\{ -8i\pi^2 k_2 F_{c\bar{c}} + 8i\pi^2 \frac{C_{h^0 \rho \tau}}{C_{h^0 \rho \tau}} [B_{23} - F_{c\bar{c}} C_{23}] + 10m_B i \pi^2 F_{c\bar{c}} + 6\gamma_5 m_B i \pi^2 F_{c\bar{c}} \right\} u_\tau(k_2)
$$

We use the completeness relation $k_2 u_\tau(k_2) = m_H u_\tau(k_2)$.

$$
M_{23} = \alpha_{23} \bar{v}_\mu(k_1) \left\{ 8i\pi^2 \frac{C_{h^0 \rho \tau}}{C_{h^0 \rho \tau}} [B_{23} - F_{c\bar{c}} C_{23} + C_{h^0 \rho \tau} (m_\tau - \frac{10}{8} m_B)] + 6\gamma_5 m_B i \pi^2 F_{c\bar{c}} \right\} u_\tau(k_2)
$$

Where the scalar and pseudoscalar parts are

$$
S_{23} = 8i\pi^2 \frac{C_{h^0 \rho \tau}}{C_{h^0 \rho \tau}} [B_{23} - F_{c\bar{c}} [C_{23} + C_{h^0 \rho \tau} (m_\tau - \frac{10}{8} m_B)]]
$$

$$
P_{23} = 6\gamma_5 m_B i \pi^2 F_{c\bar{c}}
$$

And

$$
M_{23} = \alpha_{23} \bar{v}_\mu(k_1) \{S_{23} + P_{23}\} u_\tau(k_2)
$$
For $M_{24}$

$$M_{24} = \int \frac{d^4q_1}{(2\pi)^4} \epsilon_\mu(k_1) g_{\hat{B}\hat{\mu}\tau_2} * P_{\bar{B}}(q_2) * g_{\hat{B}\tau_2\tau} * u_\tau(k_2) P_{\tau_2}(q_1) * g_{h_0\hat{\mu}\hat{\tau}_2} * P_{\mu\tau}(q_3)$$

(A.84)

However from table 3.2 we have that

$$g_{h_0\hat{\mu}\hat{\tau}_2} = 0$$

(A.85)

Therefore

$$M_{24} = 0$$

(A.86)

We write the scalar, pseudoscalar and the constant $\alpha_{24}$ as zero for simplicity

$$\alpha_{24} = 0$$
$$S_{24} = 0$$
$$P_{24} = 0$$

(A.87)
We proceed to calculate the Feynman branching ratio of the following Feynman Diagram.

\[
\begin{align*}
M_{31} &= \int \frac{d^4q_1}{(2\pi)^4} \bar{\nu}_\mu(k_1) \ast g_{\tilde{\tau}_1\mu} \ast P_{\tilde{\tau}_1}(q_2) \ast g_{\tilde{\beta}_\mu_1} \ast \tau_\mu(k_2) \ast \bar{P}_{\tilde{\beta}_\mu_1}(q_3) \ast g_{h^0\tilde{\tau}_1\tilde{\mu}_1} \ast \bar{P}_{\tilde{\tau}_1}(q_3)
\end{align*}
\]

(A.88)

However, from table 3.2 we have that the interaction of higgs $h^0$, $\tilde{\tau}_1$, $\tilde{\mu}_1$ is zero, since it does not exist in the Lagrangian. Therefore $g_{h^0\tilde{\tau}_1\tilde{\mu}_1} = 0$ and we obtain

\[
M_{31} = 0
\]

(A.89)

We write the scalar, pseudoscalar and the constant $\alpha_{31}$ as zero for simplicity

\[
\begin{align*}
\alpha_{31} &= 0 \\
S_{31} &= 0 \\
P_{31} &= 0
\end{align*}
\]

(A.90)
We can take as the fourth option the Feynman Diagram

\[
\begin{array}{c}
\text{\(\tau(k_2)\)} \\
\text{\(\tilde{\mu}_2(q_1)\)} \\
\text{\(\tilde{\tau}(q_3)\)} \\
\text{\(\tilde{\phi}(q_2)\)} \\
\text{\(\mu(k_1)\)} \\
\end{array}
\]

The Amplitude is

\[
M_{32} = \int \frac{d^4q_1}{(2\pi)^4} \tilde{\phi}_\mu(k_1) * g_{\tilde{B}\tilde{\tau}_\mu} * P_{\tilde{B}}(q_2) * g_{\tilde{B}\tilde{\tau}_2} * u_\tau(k_2) \times
\]

\[
\times P_{\tilde{B}}(q_1) * g_{h^0\tau_1\mu_2} * P_{\tau_1}(q_3)
\]

(A.91)

Obtaining \(g_{\tilde{B}\tilde{\tau}_2}, g_{\tilde{B}\tilde{\tau}_1,\mu}\) from table 3.3, we have that:

\[
g_{\tilde{B}\tilde{\tau}_1,\mu} = \frac{gs_\phi^4}{4} \tan\theta_w (1 + 3\gamma_5)
\]

(A.92)

\[
g_{\tilde{B}\tilde{\tau}_2} = -\frac{gs_\phi^4}{4} \tan\theta_w (1 + 3\gamma_5)
\]

(A.93)

Substituting the respective propagators and \(V_1, V_2\) in (A.94)

\[
M_{32} = -\int \frac{d^4q_1}{(2\pi)^4} \tilde{\phi}_\mu(k_1) \cdot \frac{gs_\phi}{4} \tan\theta_w (1 + 3\gamma_5) \cdot \frac{i(q_2 + m_\tilde{B})}{q_2^2 - m_\tilde{B}^2} \frac{gs_\phi}{4} \tan\theta_w (1 + 3\gamma_5) \cdot u_\tau(k_2) \times
\]

\[
\times \frac{i}{q_1^2 - m_\tilde{\mu}_2^2} * g_{h^0\tau_1\mu_2} \times \frac{i}{q_3^2 - m_\tilde{\tau}_1^2}
\]

\[
= \int \frac{d^4q_1}{(2\pi)^4} \tilde{\phi}_\mu(k_1) \cdot \frac{ig_\phi^4}{16} \tan^2\theta_w (1 + 3\gamma_5) \cdot \frac{q_2 + m_\tilde{B}}{q_2^2 - m_\tilde{B}^2} (1 + 3\gamma_5) \cdot u_\tau(k_2) \times
\]

\[
\times \frac{1}{q_1^2 - m_\tilde{\mu}_2^2} \times \frac{1}{q_3^2 - m_\tilde{\tau}_1^2}
\]

(A.94)

Labeling the following expressions:

\[
N_{32} = \tilde{\phi}_\mu(k_1)[1 + 3\gamma_5][q_2 + m_\tilde{B}][1 + 3\gamma_5]u_\tau(k_2)
\]

\[
\alpha_{32} = g_{h^0\tau_1\mu_2} \frac{ig_\phi^4}{16} \tan^2\theta_w
\]

\[
D_{32} = [q_2^2 - m_\tilde{B}^2][q_1^2 - m_\tilde{\mu}_2^2][q_3^2 - m_\tilde{\tau}_1^2]
\]

(A.95)
We can rewrite $M_{32}$ as

$$M_{32} = \alpha_{32} \int \frac{d^4q_1}{(2\pi)^4} N_{32} D_{32}$$  \hspace{1cm} (A.96)

It can be noticed in Eq. A.66 that $N_{32} = N_{22}$. Therefore substituing $N_{22}$ in $M_{32}$

$$M_{32} = \alpha_{32} \bar{v}_\mu(k_1) \left\{ -8k_2 \int \frac{d^4q_1}{((k_2 + q_1)^2 - m_{B}^2)(q_1^2 - m_{\mu}^2)((k_2 + k_1 + q_1)^2 - m_{\tau}^2)} ight. \left. -8 \int \frac{d^4q_1}{((k_2 + q_1)^2 - m_{B}^2)(q_1^2 - m_{\mu}^2)((k_2 + k_1 + q_1)^2 - m_{\tau}^2)} ight. 

+10m_\tilde{B} \int \frac{d^4q_1}{((k_2 + q_1)^2 - m_{B}^2)(q_1^2 - m_{\mu}^2)((k_2 + k_1 + q_1)^2 - m_{\tau}^2)} 

+6\gamma_5 m_\tilde{B} \int \frac{d^4q_1}{((k_2 + q_1)^2 - m_{B}^2)(q_1^2 - m_{\mu}^2)((k_2 + k_1 + q_1)^2 - m_{\tau}^2)} \right\} \bar{u}(k_2)$$  \hspace{1cm} (A.97)

Using the results of the integrals in Eq. 4.38 and Eq. 4.26 for this particular case (j=3, k=2)

$$M_{32} = \alpha_{32} \bar{v}_\mu(k_1) \left\{ -8i\pi^2 k_2 F_{c0} + 8 \frac{i\pi^2}{C_{h^0\mu\tau}} \{ B_{32} - F_{c0} C_{32} \} + 10m_\tilde{B} i\pi^2 F_{c0} + 6\gamma_5 m_\tilde{B} i\pi^2 F_{c0} \right\} \bar{u}(k_2)$$  \hspace{1cm} (A.98)

We use the completeness relation $\bar{k}_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$

$$M_{32} = \alpha_{32} \bar{v}_\mu(k_1) \left\{ 8 \frac{i\pi^2}{C_{h^0\mu\tau}} \{ B_{32} - F_{c0} [C_{32} + C_{h^0\mu\tau} (m_\tau - \frac{10}{8} m_\tilde{B})] \} + 6\gamma_5 m_\tilde{B} i\pi^2 F_{c0} \right\} u_\tau(k_2)$$  \hspace{1cm} (A.99)

Where the scalar and pseudoscalar parts are

$$S_{32} = 8 \frac{i\pi^2}{C_{h^0\mu\tau}} \{ B_{32} - F_{c0} [C_{32} + C_{h^0\mu\tau} (m_\tau - \frac{10}{8} m_\tilde{B})] \}$$  \hspace{1cm} (A.100)

$$P_{32} = 6\gamma_5 m_\tilde{B} i\pi^2 F_{c0}$$  \hspace{1cm} (A.101)

And

$$M_{32} = \alpha_{32} \bar{v}_\mu(k_1) \{ S_{32} + P_{32} \} u_\tau(k_2)$$  \hspace{1cm} (A.102)
We proceed to make the calculation of the amplitude $M_{33}$

$$M_{33} = \int \frac{d^4 q_1}{(2\pi)^4} \hat{v}_\mu(k_1) \ast g_{\tilde{B} \tau, \mu} \ast \hat{P}_\tilde{B}(q_2) \ast g_{\tilde{B} \tau, \tau} \ast u_\tau(k_2) \ast P_{\tau_1}(q_1) \ast g_{\nu \phi \pi, \pi} \ast P_{\tau_1}(q_3)$$

(A.103)

As we have been calculating, we take the propagators and interactions from tables 3.3, 4.3

$$g_{\tilde{B} \tau, \mu} = \frac{g s \phi}{4} \tan\theta_w [1 + 3 \gamma_5]$$

$$g_{\tilde{B} \tau, \tau} = -\frac{g c \phi}{4} \tan\theta_w [1 + 3 \gamma_5]$$

$$P_{\hat{B}}(q_2) = \frac{i(q_2 + m_{\hat{B}})}{q_2^2 - m_{\hat{B}}^2}$$

$$P_{\tau_1}(q_1) = \frac{i}{q_1^2 - m_{\tau_1}^2}$$

$$P_{\tau_1}(q_3) = \frac{i}{q_3^2 - m_{\tau_1}^2}$$

(A.104)

Therefore

$$M_{33} = -\int \frac{d^4 q_1}{(2\pi)^4} \hat{v}_\mu(k_1) \times \frac{g s \phi}{4} \tan\theta_w [1 + 3 \gamma_5] \times \frac{i(q_2 + m_{\hat{B}})}{q_2^2 - m_{\hat{B}}^2} \times \frac{g c \phi}{4} \tan\theta_w [1 + 3 \gamma_5] \times \frac{i}{q_1^2 - m_{\tau_1}^2} \times \frac{i}{q_3^2 - m_{\tau_1}^2} \times \frac{g_{\nu \phi \pi, \pi}}{16} \frac{\theta_w^2}{q_2^2 - m_{\hat{B}}^2} [1 + 3 \gamma_5] u_\tau(k_2) \frac{1}{q_1^2 - m_{\tau_1}^2} \frac{1}{q_3^2 - m_{\tau_1}^2}$$

(A.105)
APPENDIX A. EXPLICIT CALCULATION OF THE AMPLITUDES OF $H^0 \rightarrow \tau \mu$

Labeling $N_{33}, \alpha_{33}$ and $D_{33}$ as

\[ N_{33} = \bar{v}_\mu(k_1)[1 + 3\gamma_5][g_2 + m_B][1 + 3\gamma_5]u_\tau(k_2) \]

\[ \alpha_{33} = \frac{i g_{\mu\nu} r_1 g^2 c_\nu s_\omega}{16} \tan^2 \theta_w \]

\[ D_{33} = [q_2^2 - m_B^2][q_1^2 - m_{\tau_1}^2][q_3^2 - m_{\tau_1}^2] \]

(A.106)

As we can notice, the expression of $N_{33}$ is the same one to $N_{22}$ that we obtained in eq. [A.66]. Therefore we substitute the expansion of $N_{22}$ in $M_{33}$. Moreover, we substitute $q_3 = k_2 + k_1 + q_1, q_2 = k_2 + q_1$ in $D_{33}$

\[ M_{33} = \alpha_{33} \bar{v}_\mu(k_1) \left\{ -8 i k_2^2 \int \frac{d^4 q_1}{((k_2 + q_1)^2 - m_B^2)(q_1^2 - m_{\tau_1}^2)((k_2 + k_1 + q_1)^2 - m_{\tau_1}^2)} ight. 
\]

\[ -8 \int \frac{d^4 q_1}{((k_2 + q_1)^2 - m_B^2)(q_1^2 - m_{\tau_1}^2)((k_2 + k_1 + q_1)^2 - m_{\tau_1}^2)} 
\]

\[ + 10 m_B \int \frac{d^4 q_1}{((k_2 + q_1)^2 - m_B^2)(q_1^2 - m_{\tau_1}^2)((k_2 + k_1 + q_1)^2 - m_{\tau_1}^2)} 
\]

\[ + 6 \gamma_5 m_B \int \frac{d^4 q_1}{((k_2 + q_1)^2 - m_B^2)(q_1^2 - m_{\tau_1}^2)((k_2 + k_1 + q_1)^2 - m_{\tau_1}^2)} \} u_\tau(k_2) \]

(A.107)

Using the results of the integrals in Eq. 4.38 and Eq 4.26 for this particular case ($j=3, k=3$)

\[ M_{33} = \alpha_{33} \bar{v}_\mu(k_1) \left\{ -8 i \pi^2 k_2 C_{33} + 8 \frac{i \pi^2}{C_{h^0} C_{\mu}} \left\{ B_{33} - F_{\mu \tau} C_{33} \right\} + 10 m_B \pi^2 F_{\nu \tau} + 6 \gamma_5 m_B \pi^2 F_{\nu \tau} \right\} u_\tau(k_2) \]

(A.108)

We use the completeness relation $k_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$

\[ M_{33} = \alpha_{33} \bar{v}_\mu(k_1) \left\{ 8 \frac{\pi^2}{C_{h^0} C_{\mu}} \left\{ B_{33} - F_{\nu \tau} [C_{33} + C_{h^0} C_{\mu}] m_\tau - \frac{10}{8} m_B \right\} \right\} + 6 \gamma_5 m_B \pi^2 F_{\nu \tau} \} u_\tau(k_2) \]

(A.109)

Where the scalar and pseudoscalar parts are

\[ S_{33} = 8 \frac{\pi^2}{C_{h^0} C_{\mu}} \left\{ B_{33} - F_{\nu \tau} [C_{33} + C_{h^0} C_{\mu} m_\tau - \frac{10}{8} m_B] \right\} \]

(A.110)

\[ P_{33} = 6 \gamma_5 m_B \pi^2 F_{\nu \tau} \]

(A.111)

And

\[ M_{33} = \alpha_{33} \bar{v}_\mu(k_1) \left\{ S_{33} + P_{33} \right\} u_\tau(k_2) \]

(A.112)
We know that the propagators and the interactions are the following expressions, since we have extracted them from tables 3.3 and 4.3

\[
\begin{align*}
g \tilde{B} \tilde{\tau}_2 \tilde{\tau}_1 & = - \frac{g c}{4} \frac{\phi_4 \tan \theta_w [3 + \gamma_5]}{q^2 - m^2} \\
g \tilde{B} \tilde{\tau}_1 \mu & = \frac{g s}{4} \frac{\phi_4 \tan \theta_w [1 + 3 \gamma_5]}{q^2 - m^2} \\
P \tilde{B} (q_2) & = \frac{i (q_2 + m_B)}{q^2 - m^2} \\
P \tilde{\tau}_2 (q_1) & = \frac{i}{q_1 - m^2} \\
P \tilde{\tau}_1 (q_3) & = \frac{i}{q_3 - m^2}
\end{align*}
\] (A.113)

And we obtain that

\[
M_{34} = - \int \frac{d^4 q_1}{(2\pi)^4} \hat{v}_\mu (k_1) \frac{g s}{4} \frac{\phi_4 \tan \theta_w [1 + 3 \gamma_5]}{q^2 - m^2} \frac{i (q_2 + m_B)}{q^2 - m^2} \frac{gc}{4} \frac{\phi_4 \tan \theta_w [3 + \gamma_5]}{q^2 - m^2} \frac{i}{q_1 - m^2} \frac{g h_0 \tilde{\tau}_2 \tilde{\tau}_1}{q_1^2 - m^2}
\]

\[
= \int \frac{d^4 q_1}{(2\pi)^4} \hat{v}_\mu (k_1) \frac{i gh_0 \tilde{\tau}_2 \tilde{\tau}_1 g^2 c \phi_4 \tan \theta_w [1 + 3 \gamma_5]}{16} \frac{q_2^2}{q^2 - m^2} \frac{q_2^2}{q^2 - m^2} [3 + \gamma_5] u_r (k_2) \frac{1}{q_1^2 - m^2} \frac{1}{q_3^2 - m^2}
\] (A.114)

Labeling \(N_{34}, \alpha_{34}\) and \(D_{34}\) as

\[
\begin{align*}
N_{34} & = \hat{v}_\mu (k_1) [1 + 3 \gamma_5] [q_2^2 + m_B] [3 + \gamma_5] u_r (k_2) \\
\alpha_{34} & = \frac{i gh_0 \tilde{\tau}_2 \tilde{\tau}_1 g^2 c \phi_4 \tan \theta_w}{16} \\
D_{34} & = [q_2^2 - m^2_B] [q_1^2 - m^2_B] [q_3^2 - m^2_B]
\end{align*}
\] (A.115)

We can write \(M_{34} = \alpha_{34} \int \frac{d^4 q_1}{(2\pi)^4} N_{34} D_{34}\). As we can notice, the expression of \(N_{34}\) is the same one that we obtained in eq. \(A.55\). Therefore substituing Ec.
APPENDIX A. EXPLICIT CALCULATION OF THE AMPLITUDES OF $H^0 - \rightarrow \tau \mu$

\[ A.56 \] in $M_{34}$, where Ec. A.56 is $N_{34}$ expanded.

\[ M_{34} = \alpha_{34} \bar{v}_\mu(k_1) \left\{ 8 \gamma_5 \int \frac{d^4q_1}{(2\pi)^4} \frac{q_1}{[(k_2 + q_1)^2 - m_B^2][q_1^2 - m_{\tau_2}^2][(k_2 + k_1 + q_1)^2 - m_{\tau_1}^2]} \right\} \]

\[ + 8 \gamma_5 \int \frac{d^4q_1}{(2\pi)^4} (k_2 + q_1)^2 - m_{\tau_1}^2 \frac{q_1}{[(k_2 + q_1)^2 - m_{\tau_2}^2][(k_2 + k_1 + q_1)^2 - m_{\tau_1}^2]} \]

\[ + 10 \gamma_5 m_B \int \frac{d^4q_1}{(2\pi)^4} (k_2 + q_1 + k_1 + q_1)^2 - m_{\tau_1}^2 \frac{q_1}{[(k_2 + q_1)^2 - m_{\tau_2}^2][(k_2 + k_1 + q_1)^2 - m_{\tau_1}^2]} \]

\[ + 6 m_B \int \frac{d^4q_1}{(2\pi)^4} (k_2 + q_1)^2 - m_{\tau_1}^2 \frac{q_1}{[(k_2 + q_1)^2 - m_{\tau_2}^2][(k_2 + k_1 + q_1)^2 - m_{\tau_1}^2]} \} u_\tau(k_2) \]

(A.116)

We use Ec. 4.26 and Ec. 4.38 and substitute the value of the integrals above. For this particular case $j = 3$ and $k = 4$.

\[ M_{34} = \alpha_{34} \bar{v}_\mu(k_1) \left\{ -8 \gamma_5 \frac{i\pi^2}{C_{h^0\mu\tau}} (B_{34} - F_{c0} C_{34}) + 8 \gamma_5 m_\tau i\pi^2 F_{c0} + 10 \gamma_5 m_B i\pi^2 F_{c0} + 6 i\pi^2 m_B F_{c0} \right\} \]

\[ = \alpha_{34} \bar{v}_\mu(k_1) \left\{ 6 i\pi^2 m_B F_{c0} - 8 \frac{i\pi^2}{C_{h^0\mu\tau}} (B_{34} - F_{c0} [C_{34} + C_{h^0\mu\tau} (m_\tau + \frac{10}{8} m_B)]) \right\} u_\tau(k_2) \]

(A.117)

or labeling the scalar and pseudoscalar part

\[ S_{34} = \frac{6 i\pi^2 m_B F_{c0}}{C_{h^0\mu\tau}} \] \hspace{1cm} (A.118)

\[ P_{34} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} (B_{34} - F_{c0} [C_{34} + C_{h^0\mu\tau} (m_\tau + \frac{10}{8} m_B)]) \gamma_5 \]

(A.119)

\[ M_{34} = \alpha_{34} \bar{v}_\mu(k_1) \{ S_{34} + P_{34} \} \]

(A.120)
The amplitude \( M_{41} \) is calculated as follows

\[
M_{41} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_{\mu}(k_1) * g_{\tilde{B}\bar{\psi}_{\mu}} * P_{\tilde{B}}(q_2) * g_{\tilde{B}\bar{\psi}_{\mu}} * u_{\tau}(k_2) * P_{\bar{\psi}_{1}}(q_1) * g_{h_0\tilde{b}\bar{\psi}_{1}} * P_{\bar{\psi}_{3}}(q_3)
\]  
\tag{A.121}

If we see table 3.3 we have that

\[
g_{\tilde{B}\bar{\psi}_{\mu}} = \frac{g_s}{4} \tan\theta_w [3 + \gamma_5] \\
g_{\tilde{B}\bar{\psi}_{\mu}} = -\frac{g_s}{4} \tan\theta_w [3 + \gamma_5] 
\]  
\tag{A.122}

And table 4.3 give us the following expressions

\[
P_{\tilde{B}}(q_2) = \frac{i(\not{q} + m_{\tilde{B}})}{q^2 - m^2_{\tilde{B}}} \\
P_{\bar{\psi}_{1}}(q_1) = \frac{i}{q^2_{1} - m^2_{\bar{\psi}_{1}}} \\
P_{\bar{\psi}_{3}}(q_3) = \frac{i}{q^2_{3} - m^2_{\bar{\psi}_{3}}} 
\]  
\tag{A.123-125}

And we obtain

\[
M_{41} = -\int \frac{d^4q_1}{(2\pi)^4} \bar{\psi}_{\mu}(k_1) \frac{g_s}{4} \tan\theta_w [3 + \gamma_5] \frac{i(\not{q} + m_{\tilde{B}})}{q^2 - m^2_{\tilde{B}}} \frac{g_s}{4} \tan\theta_w [3 + \gamma_5] \frac{i}{q^2_{1} - m^2_{\bar{\psi}_{1}}} \frac{g_{h_0\tilde{b}\bar{\psi}_{1}}}{16} \frac{i}{q^2_{3} - m^2_{\bar{\psi}_{3}}} 
\]  
\tag{A.126}

Labeling as \( N_{41}, \alpha_{41} \) and \( D_{41} \) as

\[
N_{41} = \bar{\psi}_{\mu}(k_1) [3 + \gamma_5] \not{q} + m_{\tilde{B}} [3 + \gamma_5] u_{\tau}(k_2) \\
\alpha_{41} = \frac{-i g_{h_0\tilde{b}\bar{\psi}_{1}}}{16} \not{q}^2 \tan^2\theta_w \\
D_{41} = [\not{q}^2 - m^2_{\tilde{B}}] [q^2_{1} - m^2_{\bar{\psi}_{1}}] [q^2_{2} - m^2_{\bar{\psi}_{3}}] 
\]  
\tag{A.127}

With the expressions above we express \( M_{41} = \alpha_{41} \int \frac{d^4q_1}{(2\pi)^4} \frac{N_{41}}{D_{41}} \)
104

APPENDIX A. EXPLICIT CALCULATION OF THE AMPLITUDES OF $H^0 \rightarrow \mu \nu$

As it can be noticed $N_{41}$ is exactly as $N_{11}$ in (A.8). Therefore substituting $N_{11}$ in $M_{41}$

\[
M_{41} = \alpha_{41} \bar{v}_\mu(k_1) \left\{ 8 \int d^4q_1 \frac{q_1^4}{(2\pi)^4((q_1 + k_2)^2 - m_\mu^2)(q_1^2 - m_\mu^2)((q_1 + k_2 + k_1)^2 - m_\mu^2))} \right. \\
+ 8k_2 \int \frac{d^4q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\mu^2)((q_1 + k_2 + k_1)^2 - m_\mu^2))} + 10m_B \int \frac{d^4q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\mu^2)((q_1 + k_2 + k_1)^2 - m_\mu^2))} + 6\gamma_5 m_B \int \frac{d^4q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\mu^2)((q_1 + k_2 + k_1)^2 - m_\mu^2))} \right\} u_\tau(k_2) 
\]

We use the completeness relation that states that $k_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$

\[
M_{41} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} \left\{ \mathcal{B}_{41} = F_{\alpha_0} C_{41} \right\} + i\pi^2 F_{\alpha_0} \left\{ 8k_2 + 10m_B + 6\gamma_5 m_B \right\} 
\]

We use the completeness relation that states that $k_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$

\[
M_{41} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} \left\{ \mathcal{B}_{41} = F_{\alpha_0} [C_{41} + C_{h^0\mu\tau}(m_\tau + \frac{10}{8}m_B)] \right\} + 6i\pi^2 F_{\alpha_0} \gamma_5 \\
= \bar{v}_\mu(k_1) \alpha_{41} \left\{ S_{41} + P_{41} \right\} u_\tau(k_2) 
\]

where we called as $S_{41} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} \left\{ \mathcal{B}_{41} = F_{\alpha_0} [C_{41} + C_{h^0\mu\tau}(m_\tau + \frac{10}{8}m_B)] \right\}$ and $P_{41} = 6i\pi^2 F_{\alpha_0} \gamma_5$ because one is scalar and the second one is pseudoscalar.
The amplitude $M_{41}$ is calculated as follows

$$M_{42} = \int \frac{d^4 q_1}{(2\pi)^4} \bar{\epsilon}_\mu(k_1) * g_{\tilde{B} \tilde{\tau}_2 \mu} * P_{\tilde{B}}(q_1) * g_{\tilde{B} \tilde{\tau}_2} * u_\tau(k_2) * P_{\tilde{\tau}_2}(q_2) * g_{h_0 \tilde{\tau}_2 \tilde{\tau}_2} * P_{\tilde{\tau}_2}(q_3)$$

(A.131)

However, from table 3.3 we extract that the interaction $g_{\tilde{B} \tilde{\tau}_2 \mu} = 0$. Therefore

$$M_{42} = 0$$
$$\alpha_{42} = 0$$
$$S_{42} = 0$$
$$P_{42} = 0$$

(A.132)
In order to calculate the amplitude $M_{43}$, we use the following expression

$$M_{43} = \int \frac{d^4q_1}{(2\pi)^4} \bar{\nu}_\mu(k_1) \ast g_{B\tau_2\mu} \ast P_{\tilde{B}}(q_2) \ast g_{B\tau_1\tau} \ast u_\tau(k_2) \ast P_{\tau_1}(q_1) \ast g_{h^0\tau_2\tau_1} \ast P_{\tau_2}(q_3)$$

(A.133)

We obtain the interactions and propagators from tables 3.3 and 4.3

$$g_{B\tau_2\mu} = \frac{gs\varepsilon}{4} \tan\theta_w [3 + \gamma_5]$$
$$g_{B\tau_1\tau} = -\frac{g_c\varepsilon}{4} \tan\theta_w [1 + 3\gamma_5]$$
$$P_{\tilde{B}}(q_2) = \frac{i(q_2 + m_\tilde{B})}{q_2^2 - m_\tilde{B}^2}$$
$$P_{\tau_1}(q_1) = \frac{i}{q_1^2 - m_{\tau_1}^2}$$
$$P_{\tau_2}(q_3) = \frac{i}{q_3^2 - m_{\tau_2}^2}$$

(A.134)

And we obtain that $M_{43}$ is

$$M_{43} = -\int \frac{d^4q_1}{(2\pi)^4} \bar{\nu}_\mu \frac{gs\varepsilon}{4} \tan\theta_w [3 + \gamma_5] \frac{i(q_2 + m_\tilde{B})}{q_2^2 - m_\tilde{B}^2} \frac{g_c\varepsilon}{4} \tan\theta_w [1 + 3\gamma_5] \frac{i}{q_1^2 - m_{\tau_1}^2} \frac{i}{q_3^2 - m_{\tau_2}^2}$$

$$\times \frac{1}{q_1^2 - m_{\tau_1}^2} g_{h^0\tau_2\tau_1} \frac{1}{q_3^2 - m_{\tau_2}^2}$$

$$= -\int \frac{d^4q_1}{(2\pi)^4} \bar{\nu}_\mu \frac{ig_{h^0\tau_2\tau_1} g^2 \varepsilon^2}{16} \tan^2\theta_w [3 + \gamma_5] \frac{q_2^2 + m_\tilde{B}^2}{q_2^2 - m_\tilde{B}^2} \frac{1}{q_1^2 - m_{\tau_1}^2} \frac{1}{q_3^2 - m_{\tau_2}^2}$$

(A.135)
Labeling as $N_{43}, \alpha_{43}$ and $D_{43}$ the following expressions

\[
N_{43} = \bar{\nu}_\mu[3 + \gamma_5][q_2^2 + m_\mu][1 + 3\gamma_5]u_\tau(k_2)
\]

\[
\alpha_{43} = -i g_{\mu\rho\omega\nu} g^2 s_\rho^2 \tan^2 \theta_w
\]

\[
D_{43} = [q_2^2 - m_\mu^2][q_3^2 - m_\mu^2][q_3^2 - m_\tau^2]
\]

(A.136)

\[
N_{43} = \bar{\nu}_\mu(k_1)[3 + \gamma_5][q_2^2 + m_\mu][1 + 3\gamma_5]u_\tau(k_2)
\]

\[
\bar{\nu}_\mu(k_1)\{3(q_2^2 + m_\mu^2) + 3(q_2^2 + m_\mu^2)3\gamma_5 + \gamma_5(q_2^2 + m_\mu^2) + 5\gamma_5(q_2^2 + m_\mu^2)3\gamma_5\}u_\tau(k_2)
\]

\[
\bar{\nu}_\mu(k_1)\{3(q_2^2 + m_\mu^2) + 9\gamma_5(-q_2 + m_\mu) + 5\gamma_5(q_2 + m_\mu) + 3(-q_2 + m_\mu)\}u_\tau(k_2)
\]

(A.137)

Leaving $D_{43}$ in terms of $q_1$

\[
D_{43} = [(k_2 + q_1)^2 - m_\mu^2][q_1^2 - m_\mu^2][(k_2 + k_1 + q_1)^2 - m_\mu^2]
\]

(A.138)

Substituting Eq. A.137 A.138 in A.25

\[
M_{43} = \alpha_{12}\bar{\nu}_\mu(k_1)\left\{-8\gamma_5 \frac{d^4 q_2}{(2\pi)^4} \left[(k_2 + q_1)^2 - m_\mu^2\right][q_2^2 - m_\mu^2][(k_2 + k_1 + q_1)^2 - m_\mu^2]\right\}
\]

(A.139)

Using the result of the generalized integrals in Eq. 4.26 4.38 we obtain

\[
M_{43} = \alpha_{12}\bar{\nu}_\mu(k_1)\left\{-8i\pi^2 F_{\mu\nu}\gamma_5 \frac{d^4 q_2}{C_{\theta}^2} \left[B_{43} - F_{\mu\nu} C_{43}\right] \gamma_5
\right\}
\]

(A.140)
where we used the completeness relation $\mathfrak{k}_2 u_\tau(k_2) = m_\tau u_\tau(k_2)$ Or separating in the scalar part and pseudoscalar part

$$S_{43} = 6i m_\tilde{B} \pi^2 F_{c0}$$

$$P_{43} = \frac{8i}{C_{h^0\mu\tau}} \left\{ B_{43} - F_{c0}(C_{43} + C_{h^0\mu\tau}(m_\tau - \frac{10}{8} m_{\tilde{B}})) \right\} \gamma_5 \tag{A.141}$$

$$M_{43} = \alpha_{43} \bar{\nu}_\mu(k_1) \{ S_{43} + P_{43} \} u_\tau(k_2) \tag{A.142}$$
In order to calculate the amplitude \( M_{44} \), we use the following expression

\[
M_{44} = \int \frac{d^4 q_1}{(2\pi)^4} \bar{\nu}_\mu(k_1) * g_{\bar{B}\gamma_\mu} * P_B(q_2) * g_{\bar{B}\bar{\tau}_\tau} * u_\tau(k_2) * P_{\bar{\tau}_\tau}(q_1) * g_{\bar{\nu}_{\bar{\tau}_\tau}} * P_{\bar{\tau}_\tau}(q_3)
\]

(A.143)

Obtaining the vertexes and propagators from tables 3.3 and 4.3

\[
\begin{align*}
g_{\bar{B} \gamma_\mu} &= \frac{g s_{\psi}}{4} \tan \theta_w [3 + \gamma_5] \\
g_{\bar{B} \bar{\tau}_\tau} &= -\frac{g c_{\psi}}{4} \tan \theta_w [3 + \gamma_5] \\
P_{\bar{B}}(q_2) &= \frac{i(q_2 + m_{\bar{B}})}{q_2^2 - m_{\bar{B}}^2} \\
P_{\bar{\tau}_\tau}(q_1) &= \frac{i}{q_1^2 - m_{\bar{\tau}_\tau}^2} \\
P_{\bar{\tau}_\tau}(q_3) &= \frac{i}{q_3^2 - m_{\bar{\tau}_\tau}^2}
\end{align*}
\]

(A.144)

Therefore

\[
M_{44} = -\int \frac{d^4 q_1}{(2\pi)^4} \bar{\nu}_\mu(k_1) \frac{g s_{\psi}}{4} \tan \theta_w [3 + \gamma_5] \frac{i(q_2 + m_{\bar{B}})}{q_2^2 - m_{\bar{B}}^2} \frac{\tan \theta_w [3 + \gamma_5]}{q_2^2 - m_{\bar{B}}^2} \frac{i(q_2 + m_{\bar{B}})}{q_2^2 - m_{\bar{B}}^2} \frac{1}{q_1^2 - m_{\bar{\tau}_\tau}^2} \frac{1}{q_3^2 - m_{\bar{\tau}_\tau}^2}
\]

(A.145)

Labeling \( N_{44} \), \( \alpha_{44} \) and \( D_{44} \)

\[
\begin{align*}
N_{44} &= \bar{\nu}_\mu(k_1) [3 + \gamma_5][q_2 + m_{\bar{B}}][3 + \gamma_5]u_\tau(k_2) \\
\alpha_{44} &= -\frac{i g_{\bar{\nu}_{\bar{\tau}_\tau}} g^2 c_{\psi} s_{\psi}}{16} \tan^2 \theta_w \\
D_{44} &= [q_2^2 - m_{\bar{B}}^2][q_1^2 - m_{\bar{\tau}_\tau}^2][q_3^2 - m_{\bar{\tau}_\tau}^2]
\end{align*}
\]

(A.146)
APPENDIX A. EXPLICIT CALCULATION OF THE AMPLITUDES OF $H^0 \rightarrow \tau \mu$

With the expressions above we can write $M_{44}$ as

$$M_{44} = \alpha_{44} \bar{v}_\mu(k_1) \left\{ 8 \int d^4q_1 \frac{q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\tau^2)((q_1 + k_2 + k_1)^2 - m_\tau^2)} ight\} u_\tau(k_2)$$

$$+ 8k_2 \int \frac{d^4q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\tau^2)((q_1 + k_2 + k_1)^2 - m_\tau^2)} \right\} u_\tau(k_2)$$

$$+ 10m_B \int \frac{d^4q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\tau^2)((q_1 + k_2 + k_1)^2 - m_\tau^2)}$$

$$+ 6\gamma_5 m_B \int \frac{d^4q_1}{(2\pi)^4((q_1 + k_2)^2 - m_B^2)(q_1^2 - m_\tau^2)((q_1 + k_2 + k_1)^2 - m_\tau^2)} \right\} u_\tau(k_2)$$

(A.147)

We use the completeness relation that states that $\bar{k}_\tau u_\tau(k_2) = m_\tau u_\tau(k_2)$

$$M_{44} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} \left\{ B_{11} - F_{c0} C_{11} \right\} + i\pi^2 F_{c0} \left\{ 8k_2 + 10m_B + 6\gamma_5 m_B \right\}$$

(A.148)

We use the completeness relation that states that $\bar{k}_\tau u_\tau(k_2) = m_\tau u_\tau(k_2)$

$$M_{44} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} \left\{ B_{44} - F_{C0} [C_{44} + C_{h^0\mu\tau}(m_\tau + \frac{10}{8}m_B)] \right\} + 6i\pi^2 F_{c0} \gamma_5$$

(A.149)

where we called as $S_{44} = -8 \frac{i\pi^2}{C_{h^0\mu\tau}} \left\{ B_{44} - F_{C0} [C_{44} + C_{h^0\mu\tau}(m_\tau + \frac{10}{8}m_B)] \right\}$ and $P_{44} = 6i\pi^2 F_{c0} \gamma_5$ because one is scalar and the second one is pseudoscalar.