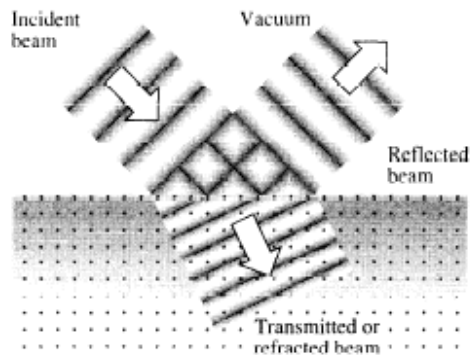
Chapter 3

Physical phenomena: plane parallel plate

This chapter provides an explanation about how rays of light physically behave when propagating through different medium (different index of refraction). In the first two chapters we have discussed how metrology instruments work and we have tried to characterize them by verifying their fabricators stated resolution and comparing them to each other. In this chapter, however, we want to understand how a beam of light propagates when going through a plane parallel plate, and a clear example of this, is a simple flat window, which we use in the tests discussed in Chapter 4. In addition to this, in this chapter we also present a numerical simulation created by Zemax that solves the plane parallel plate using the dimensions determined by the experimental setup discussed in Chapter 4. In that chapter we compare the Zemax simulation presented in this chapter to the experimental models developed in that chapter.

3.1 Snell's Law

Every time a beam of light in a certain medium interfaces another medium at some angle, each atom that composes the light is either reflected or transmitted. The transmitted beam seems to be bent, this is called refraction. The reflected beam is scattered away from the surface, this effect is called reflection. Figure 11 shows an incident beam composed of plane wavefronts impinging at some angle on a flat surface of an optically dense medium, we can observe that a certain component of the incident beam is transmitted and the other is reflected.



<u>Figure 11</u>: A lightbeam propagating through an optically dense medium, some of the beam is reflected and some is transmitted.

Let us now focus on the transmitted component; when the lightbeam penetrates the transmitted medium, the speed of the beam changes, causing the wavefronts to "bend". Let us observe Figure 12 as a multiple exposure picture that shows equal successive time intervals of a single wavefront. Δt is the time it takes point B on a wavefront to reach point D in the incident medium, with speed v_i; in the transmitted medium of that same wavefront, point A reaches point E traveling at a speed v_t. If the transmitted medium, which could be glass (n_t=1.5), is contained in air (n_i=1.003) or any other material such that $n_t > n_i$ and therefore $\overline{AE} < \overline{BD}$, the wavefront refracts (bends). The refracted wavefront extends from E to D, making a θ_t angle with the interface; in the same way, the wavefront in the incident medium, from A to B makes an angle θ_i with the interface. The triangles ABD and AED share a common hypotenuse, knowing this correlation we can say that

(1)

$$\frac{sin\theta_i}{\overline{BD}} = \frac{sin\theta_t}{\overline{AE}},$$

where the distance $\overline{BD} = v_i\Delta t$ and the segment $\overline{AE} = v_t\Delta t$. If we substitute these values in the previous equation we obtain

(2)

$$\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_t}{v_t}.$$

Since we know that $n_i = \frac{c}{v_i}$ and $n_t = \frac{c}{v_t}$, we can simply multiply both sides of the equation by c, obtaining what is known as the law of refraction or Snell's law:

(3)

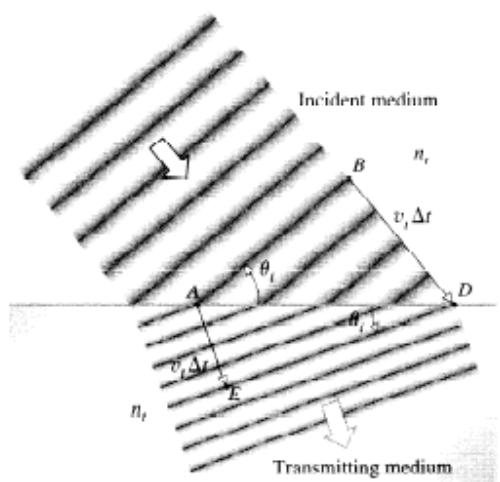
$$n_i sin\theta_i = n_t sin\theta_t.$$

We can rewrite Snell's law as the relative index of refraction of 2 different media,

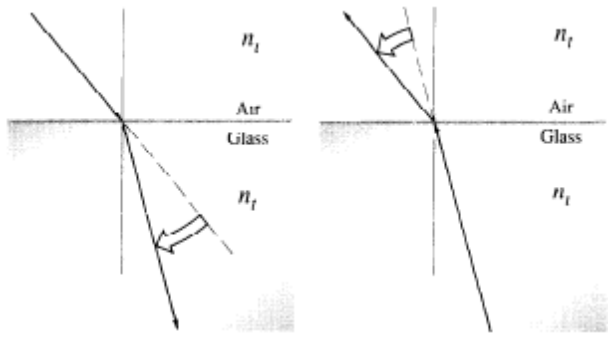
(4)

$$\frac{sin\theta_i}{sin\theta_t} = n_{ti},$$

where $n_{ti} = \frac{n_t}{n_i}$. If $n_i < n_t$, then $sin\theta_i > sin\theta_t$ simply by following Snell's law; and since the sine function is positive everywhere between 0° and 90° then $\theta_i > \theta_t$. In other words, the ray entering the medium of higher index of refraction bends towards the surface normal and viceversa, when entering a medium of lower index of refraction, the ray will bend away from the normal, as shown in Figure 13.



<u>Figure 12</u>: A lightbeam refracting because of an optically dense medium.



a) **b)**

<u>Figure 13</u>: A ray of light bending at an interface. a) From air to glass ($n_i < n_t$), the beam bends towards the perpendicular. b) From glass to air ($n_i > n_t$), the beam bends away from the perpendicular.

3.2 Plane parallel plate

When a single ray traverses a glass plate with plane surfaces parallel to each other, like a window, the ray emerges parallel to its original direction but displaced laterally a certain distance,d, which increases with the angle of incidence θ_i. Figure 14 shows a beam of light traveling through a plane parallel plate. The beam refracts at both interfaces. If we apply the law of refraction and some trigonometric functions to the propagating beam we can obtain the following relation to measure the displacement,d, such that

(5)

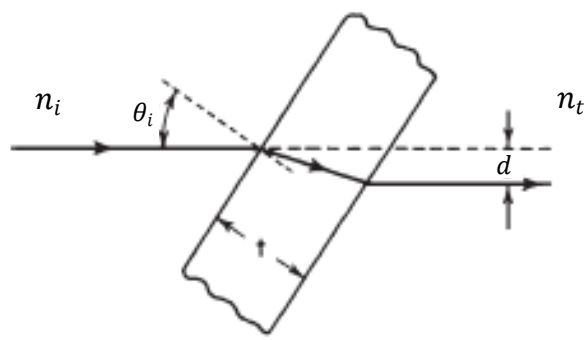
$$d = t\, sin\theta_i \left(1 - \frac{n_i}{n_t}\frac{cos\theta_i}{cos\theta_t}\right),$$

where t is the window thickness. When we have an incident angle that ranges from 0° to appreciably large angles, d is proportional to θ_i. As the angle of incidence increases, the ratio of cosines becomes noticeably less than 1, causing the parenthesis factor to increase and the d value to augment as well.

When we have small angles of incidence, equation *(5)* reduces to

$$d = t\, sin\theta_i \left(1 - \frac{n_i}{n_t}\right) \tag{6}$$

because both angles (θ_i, θ_t) are almost the same value, making the cosine ratio almost 1.



<u>Figure 14</u>: Lateral displacement d of a ray caused by a plane parallel plate.

We now understand how a plane parallel plate can introduce a displacement,d, simply by being present in an observation. It was important for us to understand this effect, because as we mentioned in the introduction of this thesis, we want to develop a correcting model for the d shift, which we will discuss in Chapter 4. However, the arrangement that we will discuss in the following chapter has the flat window vertically positioned, not tilted, just as we show in Figure 15; we mention this now because in the following section, for the Zemax numerical simulation, the window is also vertically positioned.

It is also important to mention that the d displacement depends on the incident angle, the window thickness and the indices of refraction, which also depend on the temperature, in other words

(7)

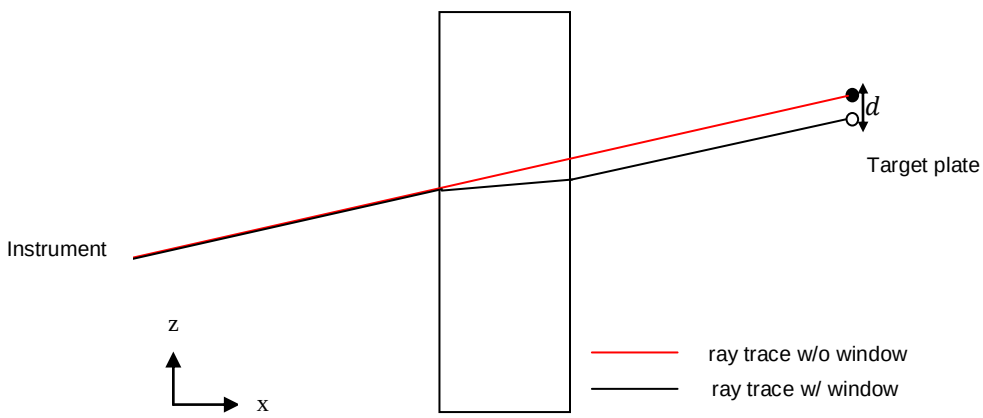
$$d = d(\theta_i, t, n_i, n_t)$$

where $n_i = n_i(T)$ and $n_t = n_t(T)$, where T is temperature. However, in this thesis, the only variable was θ_i, because the rest of the variables defined in equation *(7)* were considered to remain constant because we measured at ambient conditions and we simply wanted to solve the most simple model, allowing us to make the rest of the variables constant, turning equation *(7)* into

(8)

$$d = d(\theta_i).$$

Nonetheless, further testing will require the setup to be under vacuum, cryogenic conditions, which will make d depend on a non homogenous glass thickness, because it might curve in the middle due to the cryogens; the model will also have to consider the possible varying temperature, making it more complete and complex, just as its shown in equation *(7)*.



<u>Figure 15</u>: Vertical plane parallel plate that causes an image to shift a d distance, due to refraction.

3.3 Numerical simulation by Zemax

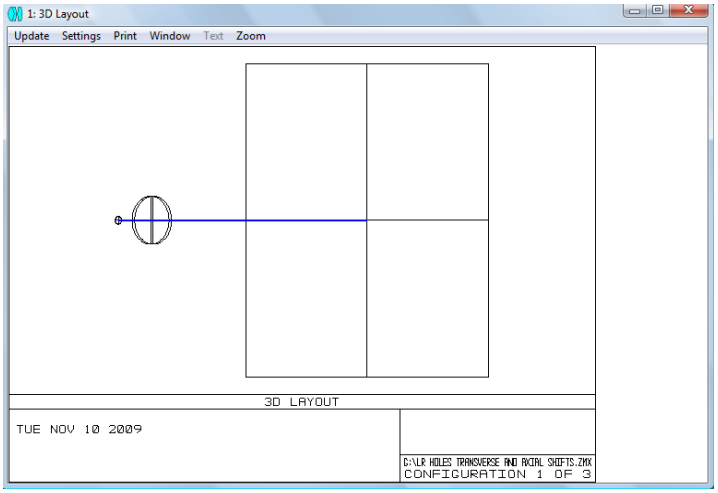
After discussing the physics behind the parallel plate effect we can now run a numerical simulation in Zemax, an optical design software capable of doing among other things, ray tracing. Let us keep in mind that in Chapter 4 we will compare the simulation presented here with the experimental results presented there.

This Zemax simulation consisted of a simple flat, wedge free window, because when we observed it with a theodolite we did not measure a wedge on it, which allowed us to consider that the window was flat.

The window material inputted for the model was BK7, a high optical quality glass, the quality is based among other things on its abbe constant. We believed BK7 to be the case because we measured the window thickness, diameter and mass, calculated its volume and then compared the measured mass (1355.5 grams) to the known value that a BK7 window with that specific volume would have (1364.8 grams). The difference between the calculated value and the one measured was less than 1%, the details of these measurements and calculations are shown in appendix B. This simple calculation allowed us to believe that the window material was most probably BK7 and therefore we could use this material for the Zemax model.

In order for us to create accurate models, we needed to have independent models for each instrument; since every instrument has its own working frequency we must create a new model for every frequency. The frequency of the laser radar, provided by Metris, was 1550nm. The frequency of the laser tracker, provided by Leica, was 780nm. Also, both instruments had their own experimental setups when measuring, consequently, even though the arrangements were the same as the ones shown in Figures 17 and 18, the distances between objects changed a few centimeters every setup. These details were required for

each model and that is why the equations shown in Table 3 will be different between instruments. However, to avoid repetition, we will only show one of the Zemax instrumental 3D layouts in Figure 16, because both look almost exactly the same.



<u>Figure 16</u>: Zemax model LR 3D layout.

The 3D Zemax layout shows a single beam of light traveling from left to right. It originated in air, then propagated through the BK7 window, perpendicular to the lens surface, exited the window and after propagating through air again, it finally formed an image on the white plane shown in Figure 16.

Zemax allows us to have different configuration models in one single design, in this case, we had 2 configuration models for each instrument, one with a window and one without. When measuring the apparent target displacement, which is the purpose of the model, it is easy to obtain the values of the shifts in the different directions using this feature. We used and excel spreadsheet to calculate the values of the shifts (see Figure 23) for the optical path length (axial direction), and the height (transverse direction).

The equations shown in Table 3 indicate the apparent shifts (called d in section 3.2) calculated by Zemax for each instrument. There are similarities between the equations, as

we would expect, because of the similar setup arrangements, but because the instruments used a different measuring frequency, the delay of the measuring beam through the window is different for each case. For both instruments, the axial shift equation is a second degree equation, because when the beam goes through the window it sags what is on the other side, like a parabola, affecting the d displacement (see equation *(5)*) parallel to the direction of the propagating beam. In the case of the transverse shift, when the beam refracts at the window surface, the shift occurs perpendicular to the propagating beam, this is the effect of a simple first degree equation. We can observe both shifts in Figure 14 if we look at the window vertically. The measured image is displaced to the side of the real image having the displacement a component in both x and z axis (see Figure 15).

Table 3. Zemax numerical simulation equations

	Axial shift	Transverse shift
LR holes and tolling balls	$y = -0.0014x^2 + 0.0005x + 12.716$	$y = 0.1516x - 0.0083$
LT SMR	$y = -0.0014x^2 + 0.0005x + 12.984$	$y = 0.1537x - 0.0083$