Appendix A[1].

Least squares adjustment

Least squares adjustment is a relatively old procedure for adjusting observations containing random errors. Karl Gauss developed it, but it was not used much until computers became more commonly used.

Spatial Analyzer uses it in many routines to provide the best and most consistent answer for fitting problems. For a group of equally weighted observations, the basic condition that is enforced in least squares adjustments is that the sum of the squares of the residuals (difference between the measured value and its most probable value) is minimized. For 3D coordinate metrology an example of a residual would be the difference between the true X-coordinate for a point and the actual X-coordinate measurement.

The method of least squares is a criterion for fitting a specified model to observed data. In this case, the specified model is the reference set of points (or expected values) and the observed data are typically the measurements or reference point values.

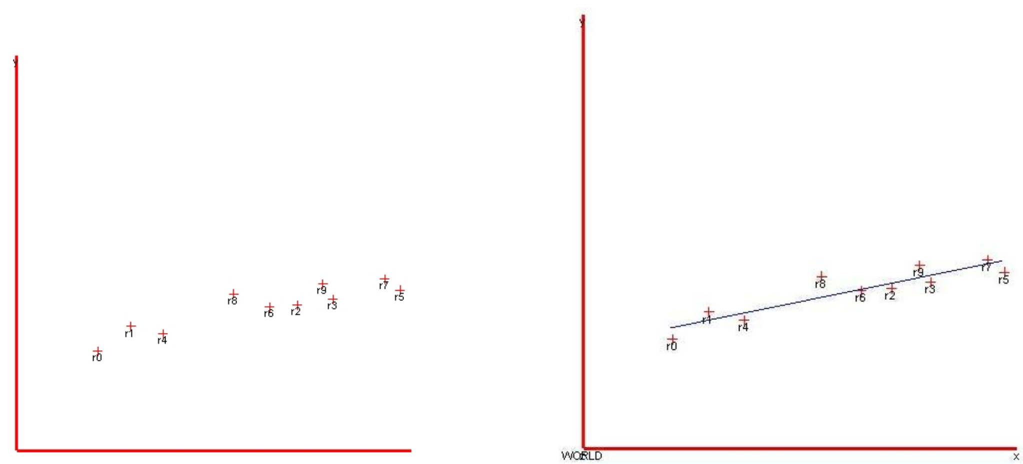
Let us present an example. The distance a car travels when driving at a particular speed is an example of a linear function. In this example, if we are trying to drive a car at a constant speed and measure how far it has gone and the time it took to get there. While trying to do all three things at the same, it is very possible that some error will bias the results. That error is generally referred to as random error. Least squares adjustment techniques are very good at dealing with this type of error and are able to produce a reasonable and consistent answer in spite of it. An example is shown in Figure 31. The time and distance are plotted on the x and y axis respectively. The points are the paired measurements made of time

[1] See reference 11.

verses the distance traveled. The distance is linear because the car is trying to maintain a constant speed.

To find what we consider the “best line” through the data, we use the least squares line routine. In this case, the residual that gets minimized is the difference squared from a measured point (Figure 31a) to the line that best describes the distance the car traveled verses the time it took. The least squares line routine will find a line, such that the sum of the distances squared from the data points to the line is the minimized. Figure 31b shows the line that best fits the points. The slope of the best-fit line is actually the average speed that the car traveled on the trip. This line is the most consistent way of determining where the car probably was at any point in time during the trip.

This ability of dealing with measurements that have at least some component of random error and still be able to produce consistent and representative answers is why we use this technique.



a) **b)**

<u>Figure 31</u>: Least square adjustment example.
a) Time vs. distance measurements. b) Least squares line fit to the measured points.