Orthogonal Polytopes: Study and Application

Doctoral Thesis presented by
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Abstract

The Extreme Vertices Model (3D-EVM) was originally presented, and widely described, by Aguilera & Ayala for representing 2-manifold Orthogonal Polyhedra (1997) and later considering both Orthogonal Polyhedra (3D-OPs) and Pseudo-Polyhedra (3D-OPPs, 1998). This model has enabled the development of simple and robust algorithms for performing the most usual and demanding tasks on solid modeling, such as closed and regularized Boolean operations, solid splitting, set membership classification operations and measure operations on 3D-OPPs. It is natural to ask if the EVM can be extended for modeling n-Dimensional Orthogonal Pseudo-Polytopes (nD-OPPs). In this sense, some experiments have been made, by Pérez-Aguila & Aguilera (2003), where the validity of the model was assumed true in order to represent 4D and 5D-OPPs.

The results previously obtained have leaded us to state, and to prove in a formal way, the Main Hypothesis of this work: The Extreme Vertices Model in the n-Dimensional Space (nD-EVM) is a complete scheme for the representation of nD-OPPs. The meaning of complete scheme is based in Requicha's set of formal criterions that every scheme must have rigorously defined: Domain, Completeness, Uniqueness and Validity (1980). The purpose of this work is to show the way the Extreme Vertices Model allows representing nD-OPPs by means of a single subset of their vertices: the Extreme Vertices. It will be seen how the Odd Edge Combinatorial Topological Characterization in the nD-OPPs has a paramount role in the foundations of the nD-EVM. Although the EVM of an nD-OPP has been defined as a subset of the nD-OPPs vertices, there is much more information about the polytope hidden within this subset of vertices. We will describe the procedures and algorithms in order to obtain this information. Finally, a set of practical applications will be discussed and modeled under the context of the EVM. In such applications we consider the representation and manipulation of 2D and 3D color animations; a method for comparing images oriented to the evaluation of volcanoes' activity; the way the nD-EVM enhances Image Based Reasoning; the extraction of useful information from "real world" 3D datasets; and finally, an application to the collision detection between 3D objects through the nD-EVM.