# Appendix H
# The "L-Shaped" nD Polytopes Family

Rucker ([Rucker77] & [Rucker84]) presents Claude Bragdon's method to define a series of well know figures which are called the **parallelotopes** [Coxeter63] or the **orthotopes** [Sommerville58]. Starting from a zero-dimensional point, Bragdon's sequence produces a segment in 1D space; a square in 2D space; a cube in 3D space; a hypercube in 4D space, and so on. See **Figure H.1**.

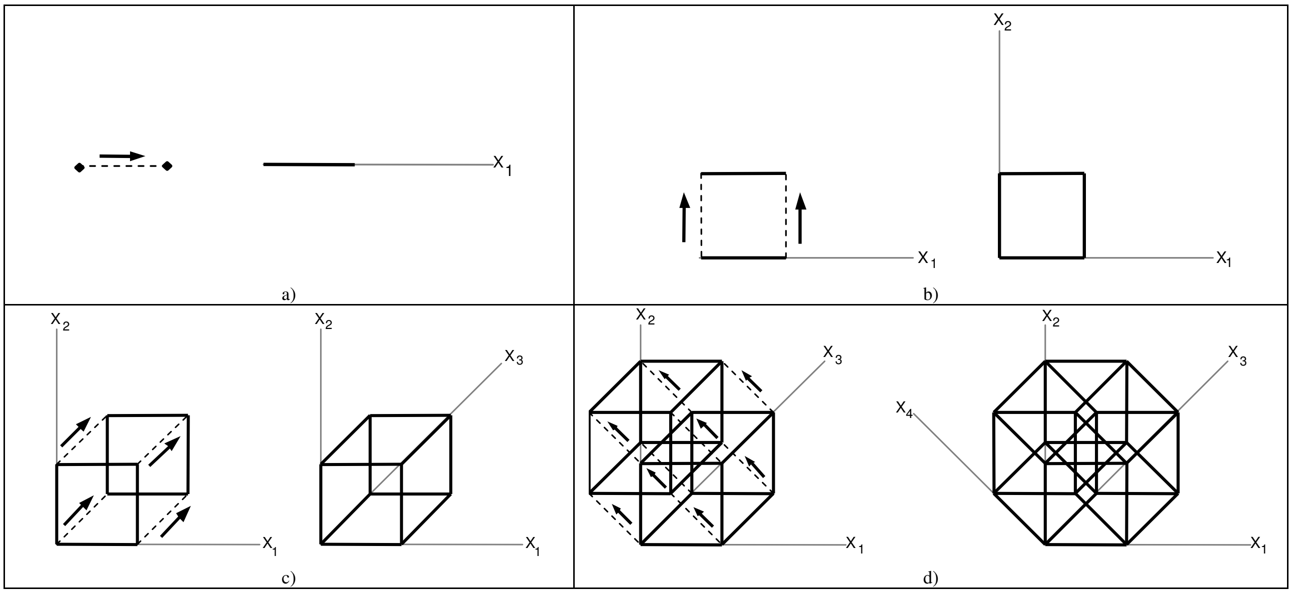


**Figure H.1.** The Claude Bragdon process for generating the 4D hypercube. a) A 1D segment is generated by the motion of a point along $X_1$-axis. b) A 2D square is generated by the motion of a segment along $X_2$-axis. c) A 3D cube is generated by the motion of a square along $X_3$-axis. d) A 4D hypercube is generated by the motion of a cube along $X_4$-axis.

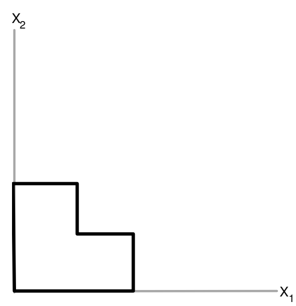


**Figure H.2.** A "L-Shaped" 2D-OPP.

The idea behind Bragdon's method can be applied in order to produce new families of nD-OPP's. In this case we will define the **"L-Shaped" family of orthogonal polytopes**. We will start by considering the "L-Shaped" 2D-OPP shown in **Figure H.2**. Such polygon is taken and moved one unit in direction of $X_3$-axis (**Figure H.3.a**). The path between the first and the second polygons produces the boundary of a new polyhedron. Such object corresponds to our new "L-Shaped" 3D-OPP (**Figure H.3.b**).


**Figure H.3.** Generation and final “L-Shaped” 3D-OPP.

Now, the “L-Shaped” 3D-OPP is moved one unit in direction of $X_4$-axis (**Figure H.4.a**). The path between the first and second new polytopes produces the 3D boundary of an “L-Shaped” 4D-OPP (**Figure H.4.b**).

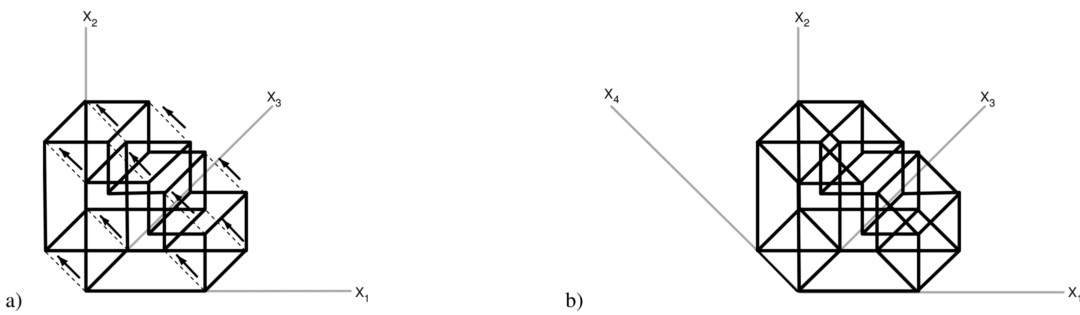


**Figure H.4.** Generation and final “L-Shaped” 4D-OPP.

As natural in Bragdon’s sequences, this process can be followed by moving our “L-Shaped” 4D-OPP one unit in direction of the fifth dimension denoted by $X_5$-axis. The path between the first and second new 4D polytopes produces the 4D boundary of an “L-Shaped” 5D-OPP. We can perform this process in order to generate the “L-Shaped” 6D-OPP, and so on.

The “L-Shaped” 4D-OPP shown in **Figure H.4.b** may be not natural to be reader in the sense that its visualization could not correspond precisely to a “*four-dimensional L*”. This is an effect due to Bragdon’s distribution of 4D space axes. By applying a 4D-3D-2D perspective projection, i.e., the polytope is projected using 4D-3D perspective projection and 3D-2D perspective projection, we obtain a visualization of the polytope which is more consistent with its name. See **Figure H.5**.

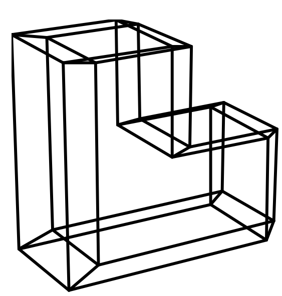

**Figure H.5.** Visualizing the “L-Shaped” 4D-OPP using 4D-3D-2D perspective projections.

By applying a 5D-4D-3D-2D perspective projection, we can visualize an “L-Shaped” 5D-OPP. See **Figure H.6.**

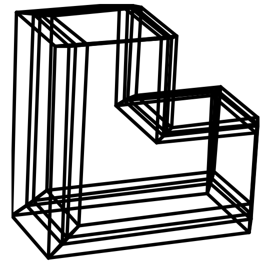

**Figure H.6.** Visualizing the “L-Shaped” 5D-OPP using 5D-4D-3D-2D perspective projections.

Another way to construct the family of polytopes we have defined is by considering the final “L-Shaped” nD-OPP as a combination of nD hyper-boxes. The rule is in fact very simple: Consider three nD hyper-boxes and distribute each one in a hyper-octant in such way that all of them are incident to a common (n-2)D cell. Such (n-2)D cell is incident to the origin of the local coordinate system described by the combination. The **Figures H.7.b**, **H.8.b** and **H.9.b** show the “L-Shaped” nD-OPP’s defined as a combination of hyper-boxes. In **Figure H.7.b** the three rectangles are incident to a same vertex which coincides with the origin; in **Figure H.8.b** the three boxes are incident to a same edge; and finally, in **Figure H.9.b** the three 4D hyper-boxes are incident to a common face. The way we have defined the construction of these combination of hyper-boxes also provides us information about the connectivity between the hyper-boxes. In **Figure H.7.a** rectangle 1 shares an edge with rectangle 2; rectangle 2 shares an edge with rectangle 3; and rectangle 3 shares a vertex with rectangle 1. In the 3D case, **Figure H.8.a** we have an analogous situation: box 1 shares a face with box 2; box 2 shares a face with box 3; and finally, box 3 shares an edge with box 1. In **Figure H.9.a** we can see that hyper-box 1 shares a volume with hyper-box 2; hyper-box 2 shares a volume with hyper-box 3; and hyper-box 3 shares a face with hyper-box 1. Hence, an “L-Shaped” nD-OPP will have the following relations between its three composing nD hyper-boxes: a first nD hyper-boxes will share a (n-1)D cell with a second nD hyper-box; such second nD hyper-box will share a (n-1)D cell with the third nD hyper-box; and finally, the third nD hyper-box will share a (n-2)D cell with the first nD hyper-box.


**Figure H.7.** The “L-shaped” 2D-OPP and the adjacencies between its composing rectangles.

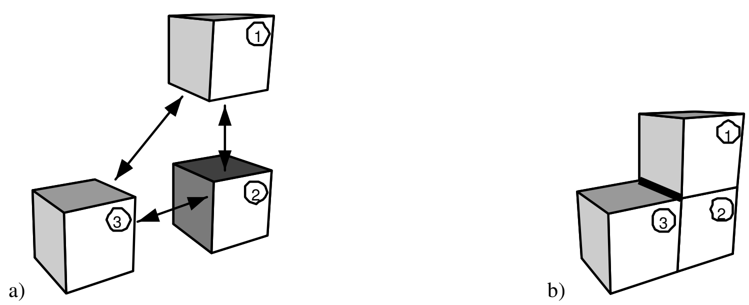


**Figure H.8.** The “L-shaped” 3D-OPP and the adjacencies between its composing boxes.

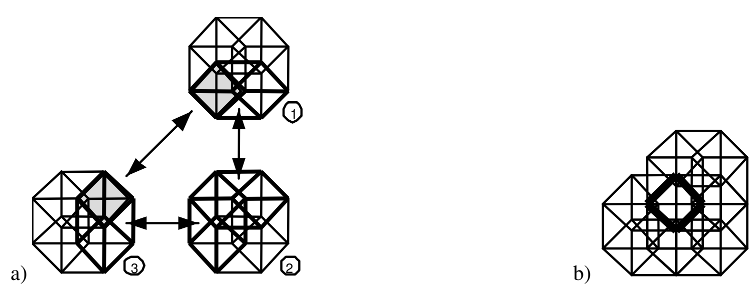


**Figure H.9.** The “L-shaped” 4D-OPP and the adjacencies between its composing hyper-boxes.