# Appendix $\mathbf{F}$ <br> Characterization of Extreme Vertices According Their Incident Odd/Even Edges and Relating Them With Other Edges' Characterizations 

Property F.1: By Property D. 1 we have that an odd edge is equivalent to a manifold edge in the 1D-OPP's. By considering such equivalence we obtain the following relation between vertices described through odd edges and vertices described through manifold edges (these last vertices were previously identified in [Aguilera98]):

| Vertices characterized by their incident odd edges <br> $\left(\frac{\text { Odd edge })}{}\right.$ | Vertices characterized by their incident manifold <br> edges $\left(\begin{array}{l}\text { Manifold edge })\end{array}\right.$ <br> Extreme Vertex <br> $\bullet$ |
| :---: | :---: |
| Extreme Vertex |  |
| 0 |  |

Property F.2: By Property D. 2 we have that an odd edge is equivalent to a manifold edge in the 2D-OPP's. Moreover, by the same property, an even edge is equivalent to a non-valid edge. By considering such equivalences we obtain the following relation between vertices described through odd/even edges and vertices described through manifold edges (these last vertices were previously identified in [Aguilera98]):

| Vertices characterized by their incident odd/even edges ( - : Odd edge, $\cdots \cdots \cdot-\cdots$ : Even edge) $)$ | Vertices characterized by their incident manifold edges ( - : Manifold edge) |
| :---: | :---: |
| Extreme Vertex | Extreme Vertex (V2) |
| Non-extreme Vertex ( $\mathrm{V}_{2,2,0}$ ) | Non-extreme Vertex (V4) |
| Non-extreme Vertex ( $\mathrm{V}_{2,1,1}$ ) | Non-extreme Vertex (Non-valid vertex) |
| Non-extreme Vertex ( $\mathrm{V}_{2,0,2}$ ) | Non-extreme Vertex (Non-valid vertex) |

Property F.3: By Property D. 3 we have that an odd edge is equivalent to a manifold edge in the 3D-OPP's. Moreover, by the same property, an even edge is equivalent to a non-manifold edge or a non-valid edge. By considering such equivalences we obtain the following relation between vertices described through odd/even edges and vertices described through manifold/non-manifold edges (these last vertices were previously identified in [Aguilera98]):

| Vertices characterized by their incident odd/even edges ( - : Odd edge, ${ }^{-\cdots-\ldots . . .}$ : Even edge) | Vertices characterized by their incident manifold/non-manifold edges ( - : Manifold edge, $-\cdots \cdot-\quad$ : Non-manifold edge) |
| :---: | :---: |
|  |  |
| Non-extreme Vertex ( $\mathrm{V}_{3,0,3}$ ) |  |
| Non-extreme Vertex ( $\mathrm{V}_{3,1,2}$ ) |  |
|  |  |
|  | Non-extreme Vertex (V6) |

Property F.4: By Property D. 4 we have that an odd edge is equivalent to an extreme edge in the 4D-OPP's. Moreover, by the same property, an even edge is equivalent to a non-extreme edge or a non-valid edge. By considering such equivalences we obtain the following relation between vertices described through odd/even edges and vertices described through extreme/non-extreme edges (these last vertices were previously identified in [Pérez-Aguila03d]):

| Vertices characterized by their incident odd/even <br>  | Vertices characterized by their incident extreme/non-extreme edges ( - : Extreme edge, : Non-extreme edge) |
| :---: | :---: |
|  | Extreme Vertices <br> VX4N2-2 |


| Vertices characterized by their incident odd/even edges ( - : Odd edge, $\cdots \cdots \cdot{ }^{-}$: Even edge) | Vertices characterized by their incident extreme/non-extreme edges ( $-\quad$ : Extreme edge, : Non-extreme edge) |
| :---: | :---: |
| Non-extreme Vertex ( $\mathrm{V}_{4,0,4}$ ) |  |
| Non-extreme Vertex $\left(\mathrm{V}_{4,1,3}\right)$ |  |
| Non-extreme Vertex $\left(\mathrm{V}_{4,2,2}\right)$ |  |
| Non-extreme Vertex $\left(\mathrm{V}_{4,3,1}\right)$ | Non-extreme Vertices |
| Non-extreme Vertex ( $\mathrm{V}_{4,4,0}$ ) | Non-Extreme Vertex (VX8) |

