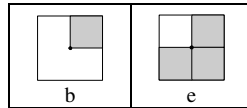


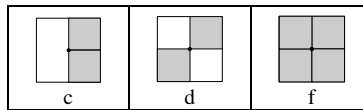
Appendix E

Some Characterizations of Extreme and Non-Extreme Vertices in the nD-OPP's

Property E.1: Consider configurations b and e for the 2D-OPP's. A configuration in the 2D-OPP's describes an extreme vertex if and only if the two 1D configurations embedded in the main axes describe extreme vertices. The segments in such 1D-OPP's are included in the boundary of the configuration.

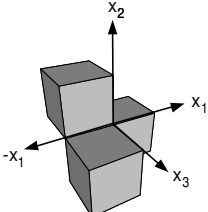
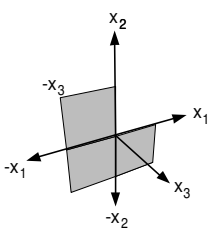
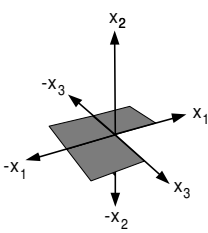
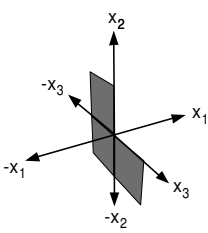
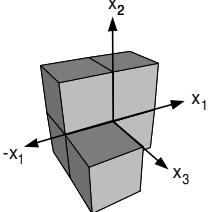
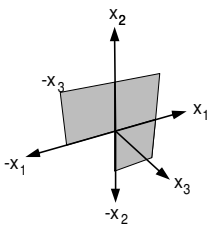
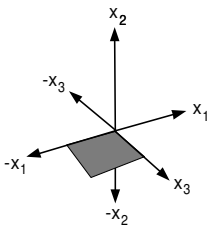
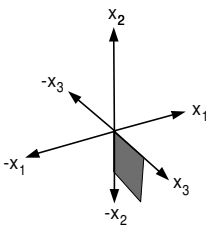
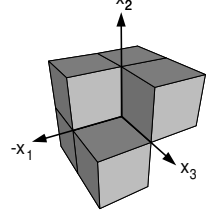
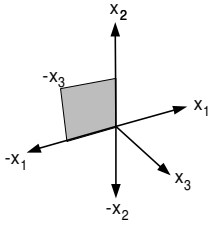
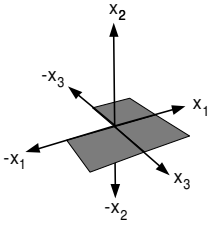
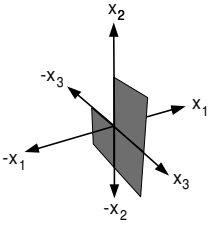
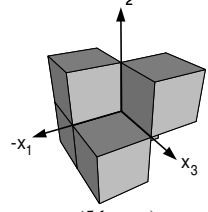
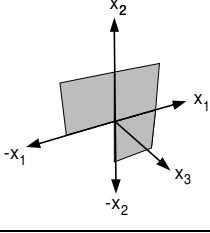
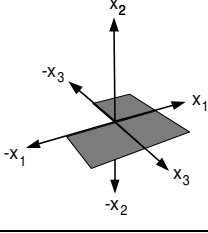
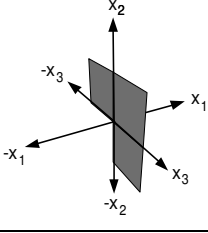
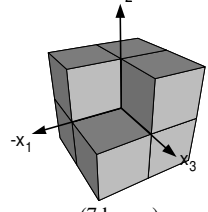
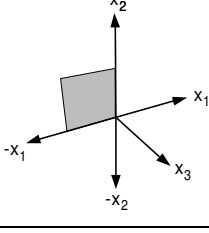
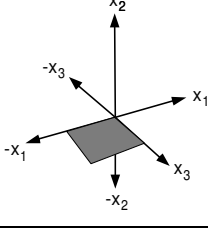
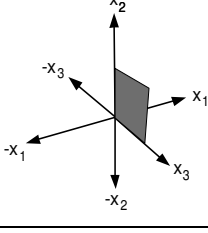


Property E.2: Consider configurations c, d and f for the 2D-OPP's. A configuration in the 2D-OPP's describes a non-extreme vertex if and only if the two 1D configurations embedded in the main axes describe non-extreme vertices. The segments in such 1D-OPP's are included in the boundary of the configuration.

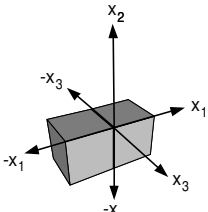
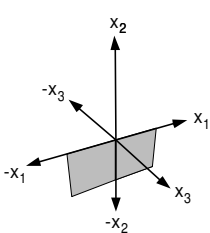
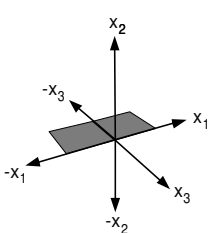
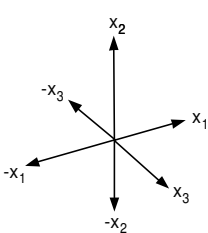


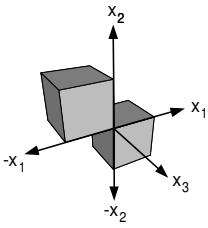
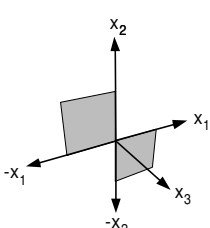
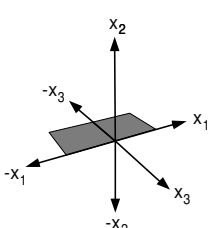
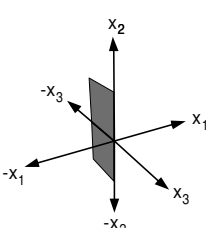
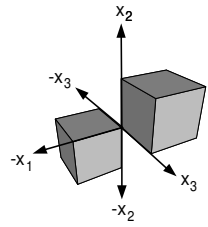
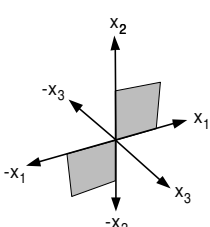
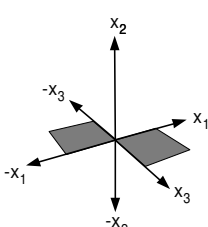
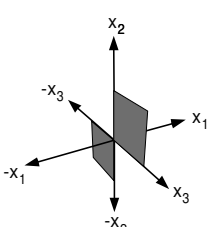
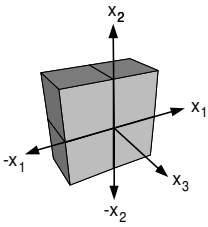
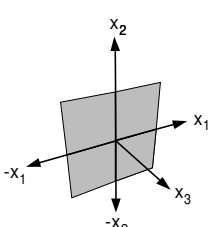
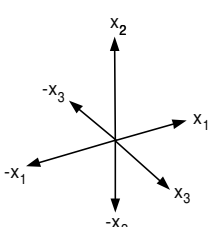
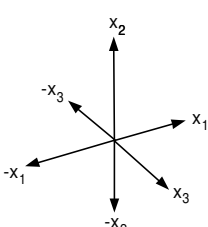
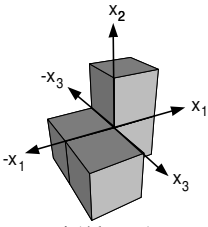
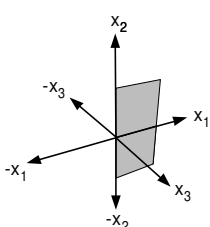
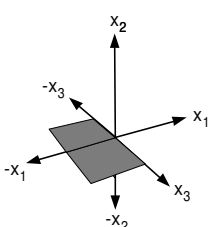
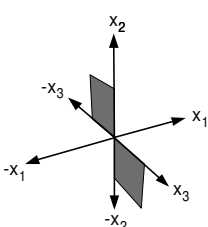
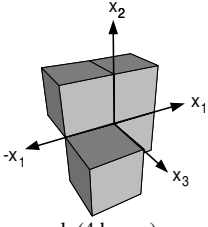
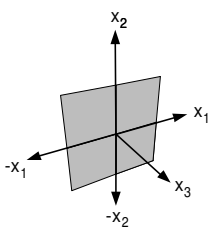
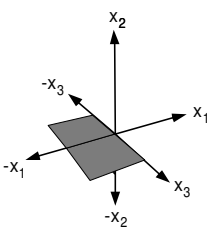
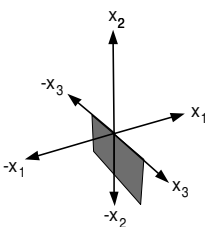
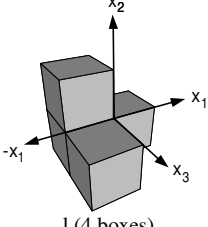
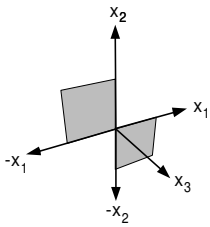
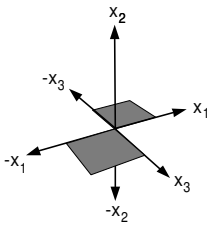
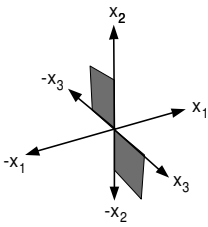
Property E.3: Consider configurations with an odd number of boxes for the 3D-OPP's. A configuration in the 3D-OPP's describes an extreme vertex if and only if the three 2D configurations embedded in the three main planes describe extreme vertices. The rectangles in such 2D-OPP's are included in the boundary of the configuration.

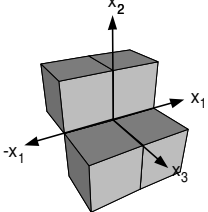
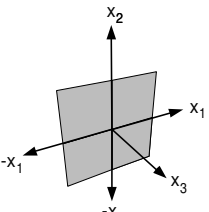
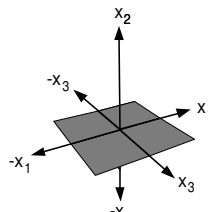
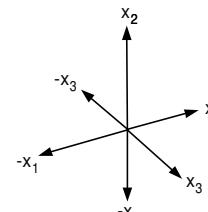
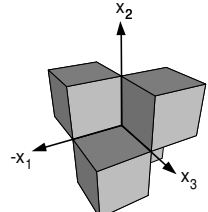
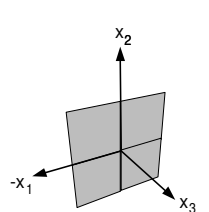
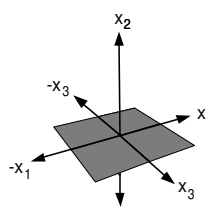
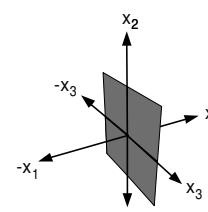
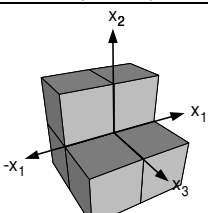
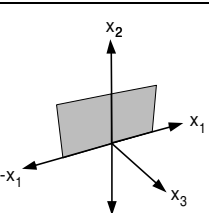
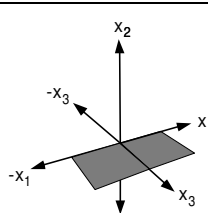
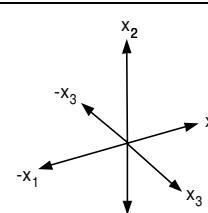
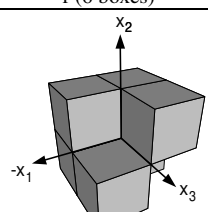
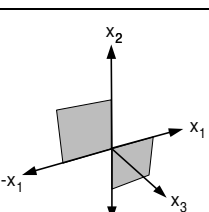
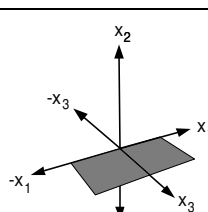
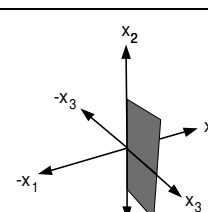
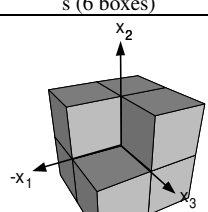
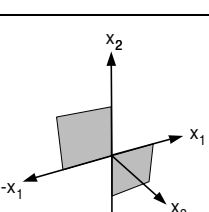
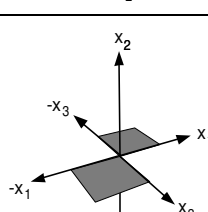
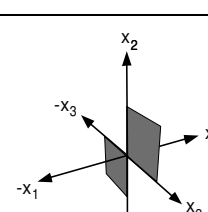
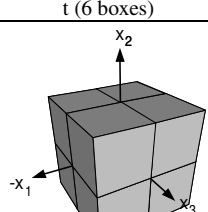
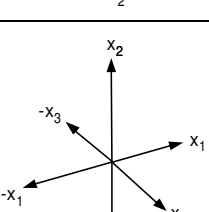
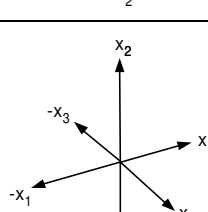
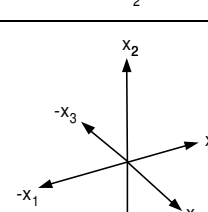
Configuration	2D-OPP in X_1X_2	2D-OPP in X_1X_3	2D-OPP in X_2X_3
<p>b (1 box)</p>			
<p>f (3 boxes)</p>			
<p>g (3 boxes)</p>			

Configuration	2D-OPP in X_1X_2	2D-OPP in X_1X_3	2D-OPP in X_2X_3
 h (3 boxes)			
 o (5 boxes)			
 p (5 boxes)			
 q (5 boxes)			
 u (7 boxes)			

Property E.4: Consider configurations for the 3D-OPP's with an even number of boxes. A configuration in the 3D-OPP's describes a non-extreme vertex if and only if the three 2D configurations embedded in the three main planes describe non-extreme vertices. The rectangles in such 2D-OPP's are included in the boundary of the configuration.

Configuration	2D-OPP in X_1X_2	2D-OPP in X_1X_3	2D-OPP in X_2X_3
 c (2 boxes)			

Configuration	2D-OPP in X_1X_2	2D-OPP in X_1X_3	2D-OPP in X_2X_3
 d (2 boxes)			
 e (2 boxes)			
 i (4 boxes)			
 j (4 boxes)			
 k (4 boxes)			
 l (4 boxes)			

Configuration	2D-OPP in X_1X_2	2D-OPP in X_1X_3	2D-OPP in X_2X_3
 m (4 boxes)			
 n (4 boxes)			
 r (6 boxes)			
 s (6 boxes)			
 t (6 boxes)			
 v (8 boxes)			

Property E.5: Consider configurations with an odd number of hyper-boxes for the 4D-OPP's. A configuration in the 4D-OPP's describes an extreme vertex if and only if the four 3D configurations embedded in the main 3D hyperplanes describe extreme vertices. The boxes in such 3D-OPP's are included in the boundary of the configuration.

			Distribution of the incident 3D volumes (not included in volume adjacency) in the four main 3D hyperplanes		
Incident 4D Hyper-boxes	Incident 3D volumes not included in volume adjacency	Hyperplane 1	Hyperplane 2	Hyperplane 3	Hyperplane 4
1	4	1	1	1	1
3	8	1	1	3	5
3	12	3	3	3	3
3	10	1	3	3	3
5	10	1	1	3	5
5	12	1	3	3	5
5	14	3	3	3	5
5	12	3	3	3	3
5	16	3	3	5	5
5	16	1	5	5	5
5	18	3	5	5	5
5	20	5	5	5	5
5	14	1	3	5	5
5	12	1	1	5	5
7	10	1	1	1	7
7	12	1	3	3	5
7	14	3	3	3	5
7	16	3	3	3	7
7	14	1	3	3	7
7	16	1	5	5	5
7	12	3	3	3	3
7	16	3	3	5	5
7	18	3	5	5	5
7	18	1	5	5	7
7	14	1	3	5	5
7	16	1	3	5	7
7	20	5	5	5	5
7	22	5	5	5	7
7	20	3	5	5	7
7	18	3	3	5	7
7	20	3	3	7	7
7	28	7	7	7	7
7	22	1	7	7	7
7	22	3	5	7	7
7	24	5	5	7	7
7	16	1	1	7	7
9	22	1	7	7	7
9	22	5	5	5	7
9	16	1	5	5	5
9	20	3	5	5	7
9	18	1	5	5	7
9	20	5	5	5	5
9	18	3	5	5	5
9	10	1	1	1	7
9	12	1	3	3	5
9	14	3	3	3	5
9	14	1	3	3	7

			Distribution of the incident 3D volumes (not included in volume adjacency) in the four main 3D hyperplanes		
Incident 4D Hyper-boxes	Incident 3D volumes not included in volume adjacency	Hyperplane 1	Hyperplane 2	Hyperplane 3	Hyperplane 4
9	16	3	3	3	7
9	12	3	3	3	3
9	16	3	3	5	5
9	14	1	3	5	5
9	18	3	3	5	7
9	16	1	3	5	7
9	16	1	1	7	7
9	22	3	5	7	7
9	24	5	5	7	7
9	28	7	7	7	7
9	20	3	3	7	7
11	10	1	1	3	5
11	12	1	1	5	5
11	12	1	3	3	5
11	14	3	3	3	5
11	12	3	3	3	3
11	16	3	3	5	5
11	20	5	5	5	5
11	14	1	3	5	5
11	16	1	5	5	5
11	18	3	5	5	5
13	10	1	3	3	3
13	8	1	1	3	3
13	12	3	3	3	3
15	4	1	1	1	1

Property E.6: Consider configurations with an even number of hyper-boxes for the 4D-OPP's. A configuration in the 4D-OPP's describes a non-extreme vertex if and only if the four 3D configurations embedded in the main 3D hyperplanes describe non-extreme vertices. The boxes in such 3D-OPP's are included in the boundary of the configuration.

			Distribution of the incident 3D volumes (not included in volume adjacency) in the four main 3D hyperplanes		
Incident 4D Hyper-boxes	Incident 3D volumes not included in volume adjacency	Hyperplane 1	Hyperplane 2	Hyperplane 3	Hyperplane 4
0	0	0	0	0	0
2	6	0	2	2	2
2	8	2	2	2	2
4	8	0	0	4	4
4	10	2	2	2	4
4	12	2	2	4	4
4	12	0	4	4	4
4	16	4	4	4	4
4	14	2	4	4	4
6	10	0	2	2	6
6	12	2	2	2	6
6	12	2	2	4	4

			Distribution of the incident 3D volumes (not included in volume adjacency) in the four main 3D hyperplanes		
Incident 4D Hyper-boxes	Incident 3D volumes not included in volume adjacency	Hyperplane 1	Hyperplane 2	Hyperplane 3	Hyperplane 4
6	14	2	4	4	4
6	16	2	4	4	6
6	16	4	4	4	4
6	18	4	4	4	6
6	14	2	2	4	6
6	18	0	6	6	6
6	20	4	4	6	6
6	18	2	4	6	6
6	14	0	2	6	6
6	16	2	2	6	6
6	24	6	6	6	6
6	20	2	6	6	6
6	22	4	6	6	6
8	24	0	8	8	8
8	20	2	6	6	6
8	22	4	6	6	6
8	26	6	6	6	8
8	22	2	6	6	8
8	24	4	4	8	8
8	32	8	8	8	8
8	16	0	0	8	8
8	12	2	2	2	6
8	14	2	2	2	8
8	12	0	4	4	4
8	12	2	2	4	4
8	14	2	4	4	4
8	16	2	4	4	6
8	8	0	0	0	8
8	14	2	2	4	6
8	16	0	4	4	8
8	20	4	4	4	8
8	20	4	4	6	6
8	18	2	4	6	6
8	16	4	4	4	4
8	18	4	4	4	6
8	16	2	2	6	6
8	18	2	2	6	8
8	24	6	6	6	6
10	12	2	2	2	6
10	10	0	2	2	6
10	12	2	2	4	4
10	14	2	2	4	6
10	14	0	2	6	6
10	16	2	2	6	6
10	16	4	4	4	4
10	18	4	4	4	6
10	14	2	4	4	4
10	16	2	4	4	6
10	20	4	4	6	6
10	18	2	4	6	6

			Distribution of the incident 3D volumes (not included in volume adjacency) in the four main 3D hyperplanes		
Incident 4D Hyper-boxes	Incident 3D volumes not included in volume adjacency	Hyperplane 1	Hyperplane 2	Hyperplane 3	Hyperplane 4
10	18	0	6	6	6
10	20	2	6	6	6
10	22	4	6	6	6
10	24	6	6	6	6
12	12	0	4	4	4
12	14	2	4	4	4
12	16	4	4	4	4
12	8	0	0	4	4
12	10	2	2	2	4
12	12	2	2	4	4
14	6	0	2	2	2
14	8	2	2	2	2
16	0	0	0	0	0