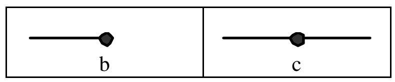
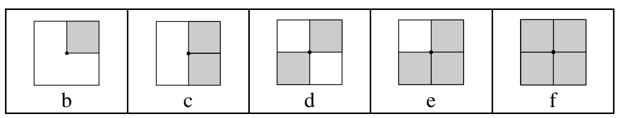
Appendix D
Some Characterizations of Odd and Even Edges in the nD-OPP's

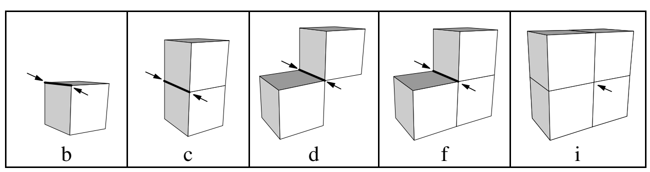
Property D.1: In 1D space an odd edge is equivalent to a manifold edge (1D-OPP's are segments and themselves are odd edges).



Property D.2: In 2D space an odd edge is equivalent to a manifold edge. In this space an even number of incident rectangles defines a non-valid edge (an edge not included in the final 2D-OPP described by the rectangles).



Property D.3: In 3D space an odd edge is equivalent to a manifold edge. In this space an even number of incident boxes defines a non-manifold edge or a non-valid edge (an edge not included in the final 3D-OPP described by the boxes).



Property D.4: In 4D space an odd edge is equivalent to an extreme edge. Moreover, the following characterizations, of odd/extreme edges in the 4D-OPP's, are identified in terms of the boundary elements (the possible 65,536 combinations of 4D hyper-boxes were exhaustively verified):

Incident 4D Hyper-boxes	Number of incident 3D volumes not included in volume adjacency
1	3
3	5
3	7
3	9
5	5
5	7
5	9
7	3

Property D.5: In the 4D-OPP's we have identified the following characterizations for odd edges (the possible 65,536 combinations of 4D hyper-boxes were exhaustively verified):

			Distribution of the incident 3D volume (not included in volume adjacency) in the three hyperplanes where the odd edge is embedded.		
Edge Classification	**Incident 4D Hyper-boxes**	**Incident 3D volumes not included in volume adjacency**	**Hyperplane 1**	**Hyperplane 2**	**Hyperplane 3**
Odd	1	3	1	1	1
Odd	3	5	1	1	3
Odd	3	9	3	3	3
Odd	3	7	1	3	3
Odd	5	5	1	1	3
Odd	5	7	1	3	3
Odd	5	9	3	3	3
Odd	7	3	1	1	1

Property D.6: In the 4D-OPP's we have identified the following characterizations for even edges (the possible 65,536 combinations of 4D hyper-boxes were exhaustively verified):

			Distribution of the incident 3D volumes (not included in volume adjacency) in the three hyperplanes where the even edge is embedded.		
Edge Classification	**Incident 4D Hyper-boxes**	**Incident 3D volumes not included in volume adjacency**	**Hyperplane 1**	**Hyperplane 2**	**Hyperplane 3**
Even	0	0	0	0	0
Even	2	4	0	2	2
Even	2	6	2	2	2
Even	4	4	0	0	4
Even	4	6	2	2	2
Even	4	8	2	2	4
Even	4	8	0	4	4
Even	4	12	4	4	4
Even	6	4	0	2	2
Even	6	6	2	2	2
Even	8	0	0	0	0

Property D.7: In the 5D-OPP's we have identified the following characterizations for odd edges (the possible 2^{32} = 4,294,967,296 combinations of 5D hyper-boxes were exhaustively verified):

			Distribution of the incident 4D hypervolumes (not included in 4D hypervolume adjacency) in the four hyperplanes where the odd edge is embedded.			
Edge Classification	**Incident 5D Hyper-boxes**	**Incident 4D hypervolumes not included in 4D hypervolume adjacency**	**Hyperplane 1**	**Hyperplane 2**	**Hyperplane 3**	**Hyperplane 4**
Odd	1	4	1	1	1	1
Odd	3	8	1	1	3	3
Odd	3	10	1	3	3	3
Odd	3	12	3	3	3	3
Odd	5	10	1	1	3	5
Odd	5	12	1	3	3	5
Odd	5	12	3	3	3	3
Odd	5	12	1	1	5	5
Odd	5	14	3	3	3	5
Odd	5	14	1	3	5	5
Odd	5	16	3	3	5	5
Odd	5	16	1	5	5	5
Odd	5	18	3	5	5	5
Odd	5	20	5	5	5	5
Odd	7	10	1	1	1	7

Edge Classification	Incident 5D Hyper-boxes	Incident 4D hypervolumes not included in 4D hypervolume adjacency	Distribution of the incident 4D hypervolumes (not included in 4D hypervolume adjacency) in the four hyperplanes where the odd edge is embedded.			
			Hyperplane 1	**Hyperplane 2**	**Hyperplane 3**	**Hyperplane 4**
Odd	7	12	1	3	3	5
Odd	7	12	3	3	3	3
Odd	7	14	3	3	3	5
Odd	7	14	1	3	3	7
Odd	7	14	1	3	5	5
Odd	7	16	1	3	5	7
Odd	7	16	3	3	3	7
Odd	7	16	1	5	5	5
Odd	7	16	3	3	5	5
Odd	7	16	1	1	7	7
Odd	7	18	3	5	5	5
Odd	7	18	1	5	5	7
Odd	7	18	3	3	5	7
Odd	7	20	5	5	5	5
Odd	7	20	3	5	5	7
Odd	7	20	3	3	7	7
Odd	7	22	5	5	5	7
Odd	7	22	1	7	7	7
Odd	7	22	3	5	7	7
Odd	7	24	5	5	7	7
Odd	7	28	7	7	7	7
Odd	9	10	1	1	1	7
Odd	9	12	1	3	3	5
Odd	9	12	3	3	3	3
Odd	9	14	1	3	3	7
Odd	9	14	3	3	3	5
Odd	9	14	1	3	5	5
Odd	9	16	3	3	3	7
Odd	9	16	3	3	5	5
Odd	9	16	1	3	5	7
Odd	9	16	1	1	7	7
Odd	9	16	1	5	5	5
Odd	9	18	3	3	5	7
Odd	9	18	3	5	5	5
Odd	9	18	1	5	5	7
Odd	9	20	5	5	5	5
Odd	9	20	3	5	5	7
Odd	9	20	3	3	7	7
Odd	9	22	1	7	7	7
Odd	9	22	5	5	5	7
Odd	9	22	3	5	7	7
Odd	9	24	5	5	7	7
Odd	9	28	7	7	7	7
Odd	11	10	1	1	3	5
Odd	11	12	1	1	5	5
Odd	11	12	3	3	3	3
Odd	11	12	1	3	3	5
Odd	11	14	1	3	5	5
Odd	11	14	3	3	3	5
Odd	11	16	3	3	5	5
Odd	11	16	1	5	5	5
Odd	11	18	3	5	5	5
Odd	11	20	5	5	5	5
Odd	13	8	1	1	3	3
Odd	13	10	1	3	3	3
Odd	13	12	3	3	3	3
Odd	15	4	1	1	1	1

Property D.8: In the 5D-OPP's we have identified the following characterizations for even edges (the possible 2^{32} = 4,294,967,296 combinations of 5D hyper-boxes were exhaustively verified):

			Distribution of the incident 4D hypervolumes (not included in 4D hypervolume adjacency) in the four hyperplanes where the even edge is embedded.			
Edge Classification	**Incident 5D Hyper-boxes**	**Incident 4D hypervolumes not included in 4D hypervolume adjacency**	**Hyperplane 1**	**Hyperplane 2**	**Hyperplane 3**	**Hyperplane 4**
Even	0	0	0	0	0	0
Even	2	6	0	2	2	2
Even	2	8	2	2	2	2
Even	4	8	0	0	4	4
Even	4	10	2	2	2	4
Even	4	12	2	2	4	4
Even	4	12	0	4	4	4
Even	4	14	2	4	4	4
Even	4	16	4	4	4	4
Even	6	10	0	2	2	6
Even	6	12	2	2	2	6
Even	6	12	2	2	4	4
Even	6	14	2	4	4	4
Even	6	14	2	2	4	6
Even	6	14	0	2	6	6
Even	6	16	2	2	6	6
Even	6	16	2	4	4	6
Even	6	16	4	4	4	4
Even	6	18	4	4	4	6
Even	6	18	0	6	6	6
Even	6	18	2	4	6	6
Even	6	20	4	4	6	6
Even	6	20	2	6	6	6
Even	6	22	4	6	6	6
Even	6	24	6	6	6	6
Even	8	8	0	0	0	8
Even	8	12	2	2	2	6
Even	8	12	0	4	4	4
Even	8	12	2	2	4	4
Even	8	14	2	4	4	4
Even	8	14	2	2	2	8
Even	8	14	2	2	4	6
Even	8	16	2	4	4	6
Even	8	16	0	4	4	8
Even	8	16	4	4	4	4
Even	8	16	2	2	6	6
Even	8	16	0	0	8	8
Even	8	18	2	2	6	8
Even	8	18	2	4	6	6
Even	8	18	4	4	4	6
Even	8	20	4	4	4	8
Even	8	20	4	4	6	6
Even	8	20	2	6	6	6
Even	8	22	4	6	6	6
Even	8	22	2	6	6	8
Even	8	24	6	6	6	6
Even	8	24	4	4	8	8
Even	8	24	0	8	8	8
Even	8	26	6	6	6	8
Even	8	32	8	8	8	8
Even	10	10	0	2	2	6
Even	10	12	2	2	2	6
Even	10	12	2	2	4	4
Even	10	14	2	2	4	6
Even	10	14	0	2	6	6

Edge Classification	Incident 5D Hyper-boxes	Incident 4D hypervolumes not included in 4D hypervolume adjacency	Distribution of the incident 4D hypervolumes (not included in 4D hypervolume adjacency) in the four hyperplanes where the even edge is embedded.			
			Hyperplane 1	**Hyperplane 2**	**Hyperplane 3**	**Hyperplane 4**
Even	10	14	2	4	4	4
Even	10	16	2	4	4	6
Even	10	16	2	2	6	6
Even	10	16	4	4	4	4
Even	10	18	4	4	4	6
Even	10	18	2	4	6	6
Even	10	18	0	6	6	6
Even	10	20	2	6	6	6
Even	10	20	4	4	6	6
Even	10	22	4	6	6	6
Even	10	24	6	6	6	6
Even	12	8	0	0	4	4
Even	12	10	2	2	2	4
Even	12	12	2	2	4	4
Even	12	12	0	4	4	4
Even	12	14	2	4	4	4
Even	12	16	4	4	4	4
Even	14	6	0	2	2	2
Even	14	8	2	2	2	2
Even	16	0	0	0	0	0