# Chapter 4

# Mathematical description of the physical requirements to place a CAVE

# 4. Mathematical description of the physical requirements to place a CAVE

The building of virtual environments can be carried out using a variety of output devices; one of the most common devices is the projector, because this tool allows us to build big environments. Nevertheless, the projector has the disadvantage when the environment is bigger, because more space is required. The projector needs an extra distance to screen an image as well as the required space for the virtual environment. The used distance for a projector goes from the projector to the wall where the image is to be screened.

In this chapter, a CAVE has been used as a model for computing its required space according its position and location in a room. Although, this research focuses on the CAVE model, the ideas of this chapter can be applied in any other virtual environments built with projectors. In chapter 5 there is a virtual environment where this technique has been applied. The result of this research is published in the Research Journal in Computing Science (Mora, et al., 2008-4).

This chapter is organized as follows:

Section 4.1 describes some characteristics about projectors.

Section 4.2 presents the formulae for determining the size of the room where a CAVE should be built.

Section 4.3 shows a system, which uses the equations presented in section 4.2, furthermore in this section different configurations are shown.

In section 4.4 there is a chapter summary.

## 4.1 Tests with Projectors

The projectors have been very useful tools in the building of virtual environments; one reason is that the same projector allows for displaying an image in different sizes. Therefore, it is possible to build an environment with dimensions $3m \times 3m \times 3m$ or any other size using the same projectors.

A set of tests has been carried out for determining the relation between the size of the screened image and the distance from the projector as far as the wall on which the image would be displayed. For these tests, different brands of projectors were used, but the results were similar. Table 4.1 shows different examples of projection size and are used in Figure 4.1 to show the maximum screening size when the distance from the projector to the wall is 3m. It is possible to conclude that in general the width of the screened image is almost half of this distance. For this reason, in the rest of this work the screening width will be taken as a half of this distance. Each screening has an aspect ratio 4:3.

**Table 4.1 Examples of screening size**

| Distance between projector and screen | Width of the screened image | Height of the screened image |
|---|---|---|
| 2m | 1.07m | 0.8m |
| 2.3m | 1.23m | 0.92m |
| 2.5m | 1.33m | 1m |
| 2.7m | 1.44m | 1.08m |
| 3m | 1.6m | 1.2m |
| 3.3m | 1.76m | 1.32m |
| 3.5m | 1.87m | 1.4m |
| 3.7m | 2.97m | 1.48m |
| 4m | 2.13m | 1.6m |

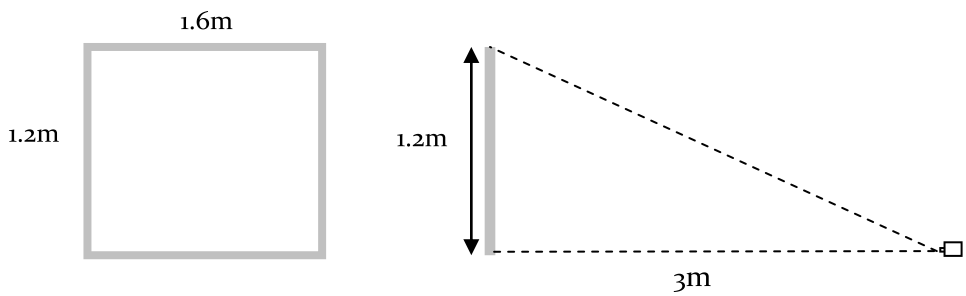


**Figure 4.1 Image size obtained at a distance of 3 m.**

## 4.2 Room Size

The plan of a room where a CAVE may be built could determine the dimensions of this CAVE, but, the way in how the screens and other devices are distributed in the room allow us to take advantage of the space.

In order to achieve a good immersion level within the CAVE, its walls have to be higher than the average height of man, so people do not have the opportunity to look over them. Using a screen height of 2.10 m. should be enough for most situations. Considering the aspect ratio of 4:3, would mean that the screen height is 75% of its width. This document names the screen width as *P*. In futures examples of this chapter *P* = 2.80 m.

### 4.2.1 Room design #1

Figure 4.2 shows the first attempt. The small squares represent projectors, the black lines from the projector represent the screening, the thick black lines represent screens, and the thick gray lines represent room walls. This design has three screens, every screen is parallel to a wall of the room, and the projectors are placed behind the screens. The walls have been named as *W1* and *W2*.

Eq. 4.1 and Eq. 4.2 show the length corresponding to the walls. Eq. 4.3 determines the dimension of the Room.

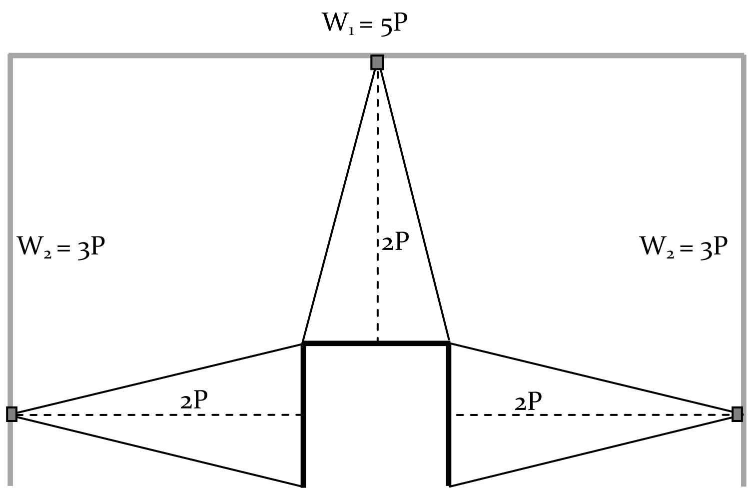

**Figure 4.2 The CAVE screens are parallel to the room walls**

**Eq. 4.1** $$W_1 = 5P$$

**Eq. 4.2** $$W_2 = 3P$$

**Eq. 4.3** $$A_1 = (5P)(3P) = 15P^2$$

This means that the room width should be at least $5P$, while its length should be larger than $3P$ to allow the user to enter into the cave.

Using $P = 2.80$ *m.*, the room size should be: $A_1 = 117.6\ m^2$.

### 4.2.2 Room design #2

This section presents the second attempt. In order to reduce the room size, the projectors can be set at the corners of it, the screens will be turned 45 degrees compared with the first design.

In Figure 4.3 the small squares represent projectors, the black lines from projector represent the screening, the thick black lines represent screens, and the thick gray lines represent room walls. This figure shows that the distance from the center of the room to any corner is: $5P/2$. Pythagorean Theorem is used for calculating the length of each wall, which is: $W = 5P\sqrt{2}/2$.

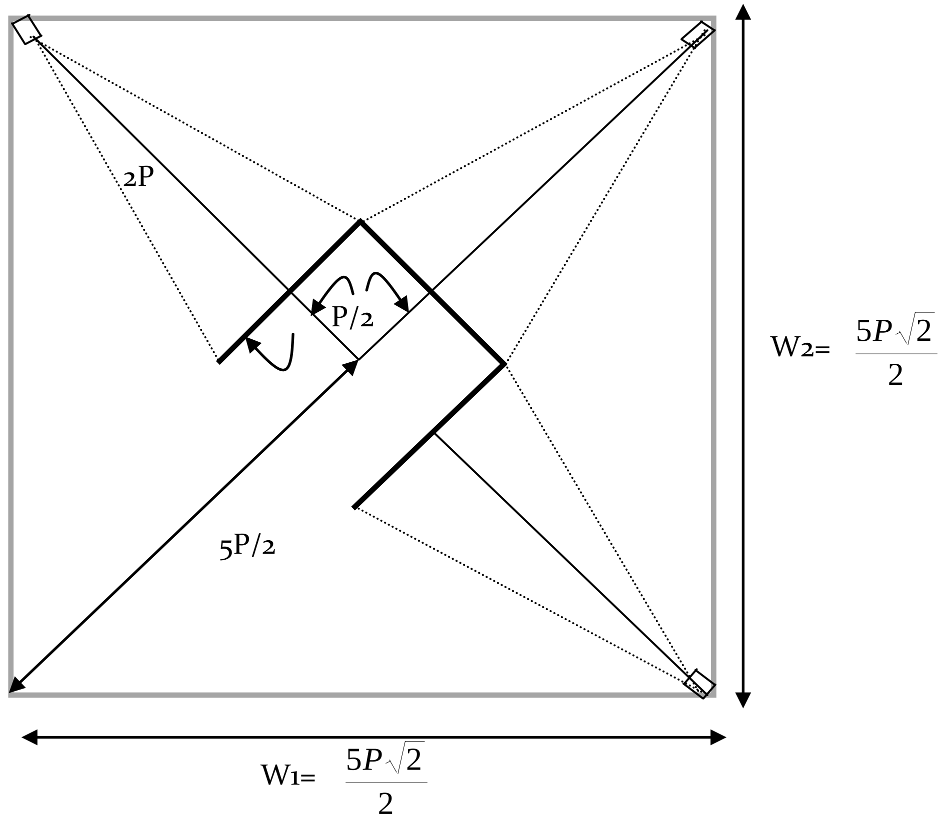


**Figure 4.3 Shows the room with the projectors placed at the corners**

Eq. 4.4 determines the dimensions of the room.

**Eq. 4.4**
$$A_2 = \left(\frac{5P\sqrt{2}}{2}\right)\left(\frac{5P\sqrt{2}}{2}\right) = \frac{25P^2}{2}$$

Using $P$=2.80 $m$., the room size is $A\_2$=98$m^2$

### 4.2.3 Room design #3

As it was previously mentioned, the projection size depends on the distance between the projector and the wall, nevertheless a set of mirrors can help to reduce the requirements of the room. The projector reflects an image on a mirror, the mirror reflects the same image on a screen, but inverted laterally. Using this strategy the projectors can be placed nearer to the screens. With this technique the distance used for screening and the screening width are both equal to $P$. Figure 4.4 shows the size of the obtained screening when a mirror is used

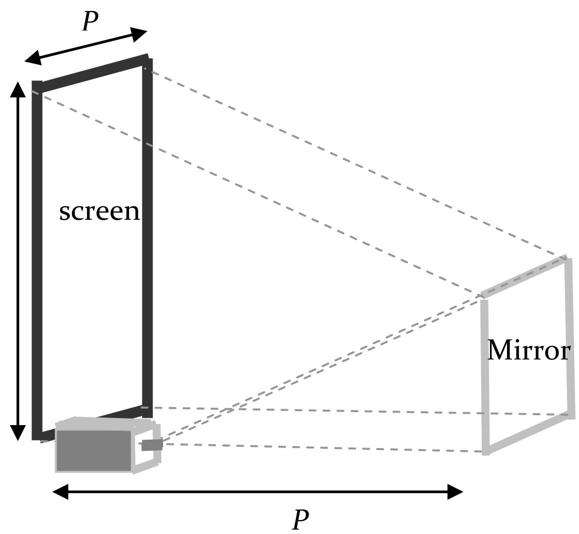


**Figure 4.4 shows the projection width using mirrors**

Figure 4.5 shows the third attempt. This figure allows us to see that the distance from the center of the room to any corner is: $\frac{7P}{4}$ Pythagorean Theorem is used for calculating the length of each wall, which is $\frac{7P\sqrt{2}}{4}$. In this design, the mirror width is half the screens size, that is $\frac{P}{2}$

Eq. 4.5 shows the room size of the third design:

**Eq. 4.5** $$A_3 = \left(7P\frac{\sqrt{2}}{4}\right)\left(7P\frac{\sqrt{2}}{4}\right) = \frac{49P^2}{8}$$

The room size for our example, where $P$ = 2.80m., should be $A\_3$= 48.02$m^2$

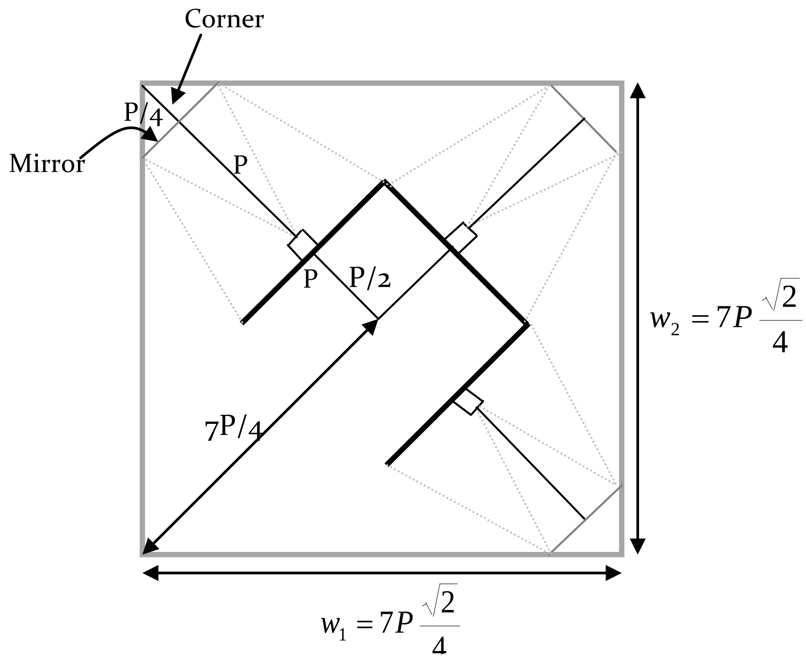


**Figure 4.5 Room size using mirrors**

## 4.2.4 Room design #4

It is possible to provide a better distribution of the mirrors and the projectors. Figure 4.6 shows a mirror turned with respect to the projector. On turning the mirror some degrees, the image will be projected in a direction, which will depend of angle turned. Specifically, the direction of the projection has an angle twice as big as the mirror's angle, for example, if we rotate the mirror $45^{o}$ the projection will rotate $90^{o}$.

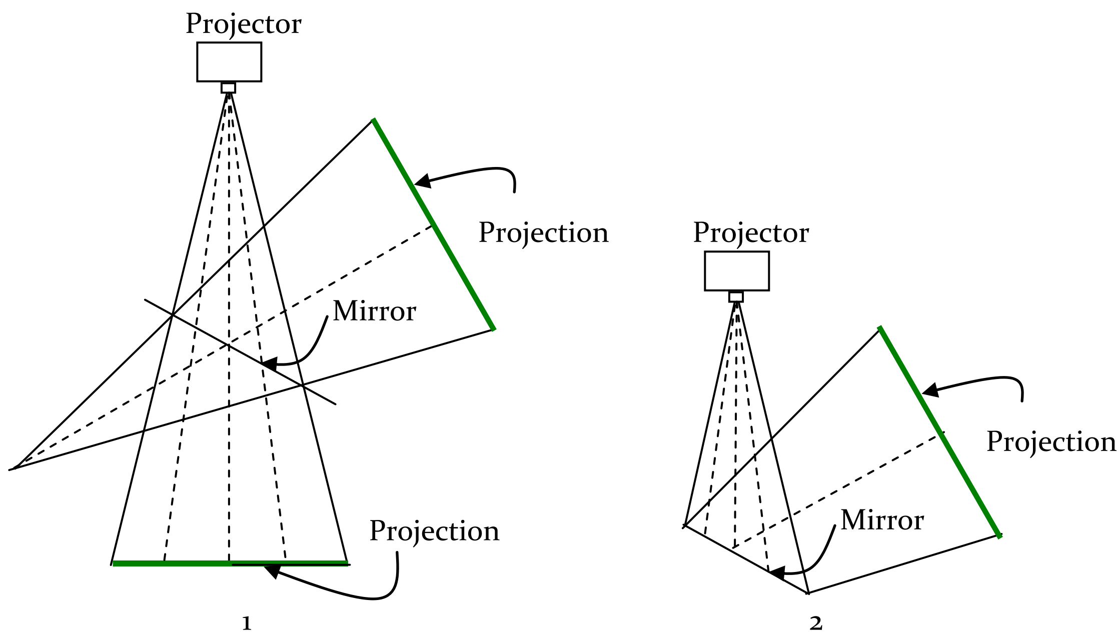


**Figure 4.6 Projection using a turned mirror.**

Nevertheless, it is necessary to find the best way to turn the mirror, because it is possible that the projection crosses some screen of the environment or overlaps it. See Figure 4.7

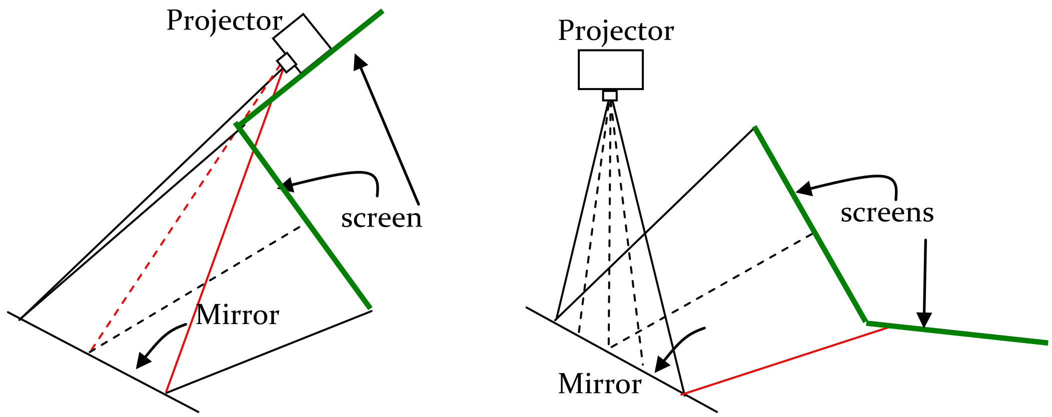


**Figure 4.7 Problems using a mirror**

In this configuration the mirror size ( $\mathrm{m_{width}} \times \mathrm{m_{heigth}}$) depends on the distance (*d*) between it and the screen, as well as its rotation ( $\theta$ ), see Figure 4.8.1. The mirror size is computed by splitting in two the triangle formed between the position of the virtual projector and the mirror, see Figure 4.8.2 and equations 4.6 to 4.14.

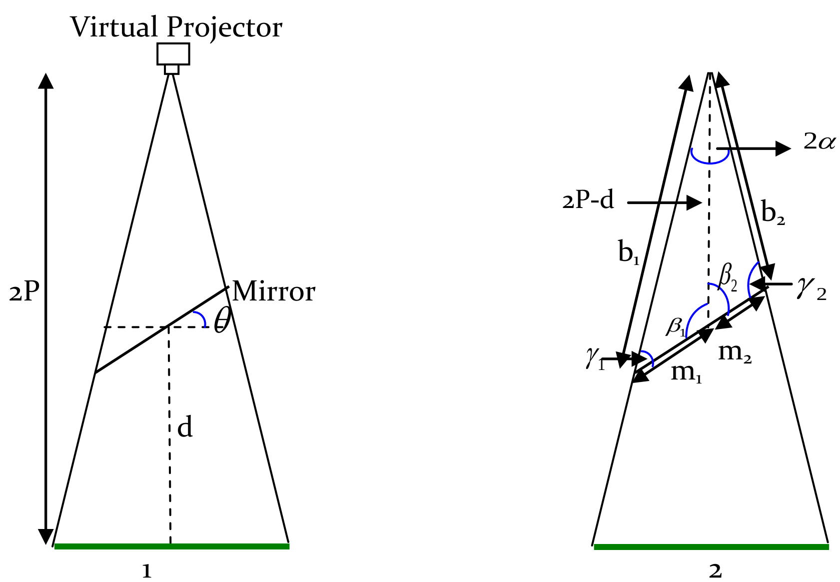


**Figure 4.8 Mirror and screen**

**Eq. 4.6** $$\alpha = a\tan\left(\frac{1}{4}\right)$$

**Eq. 4.7** $$\beta_1 = \frac{\pi}{2} + \theta$$

**Eq. 4.8** $$\gamma_1 = \pi - \alpha - \beta_1$$

**Eq. 4.9** $\beta_2 = \frac{\pi}{2} - \theta$

**Eq. 4.10** $\gamma_2 = \pi - \alpha - \beta_2$

**Eq. 4.11** $m_1 = \frac{(2P-d)\sin(\alpha)}{\sin(\gamma_1)}$

**Eq. 4.12** $m_2 = \frac{(2P-d)\sin(\alpha)}{\sin(\gamma_2)}$

**Eq. 4.13** $m_{width} = m_1 + m_2$

**Eq. 4.14** $m_{height} = \frac{3m_{width}}{4}$

Where *d* and $\theta$ are changeable.

This design uses the mirror and projector technique, as well as a rotation between the CAVE and the room walls, which allow reducing the space. Figure 4.9 shows the design number four. The CAVE center has the coordinates *x,y*. In this design is proposed that $d_1$, $d_2$,, *and* $d_3$ could be different, as well as $\theta_1, \theta_2$, and $\theta_3$.

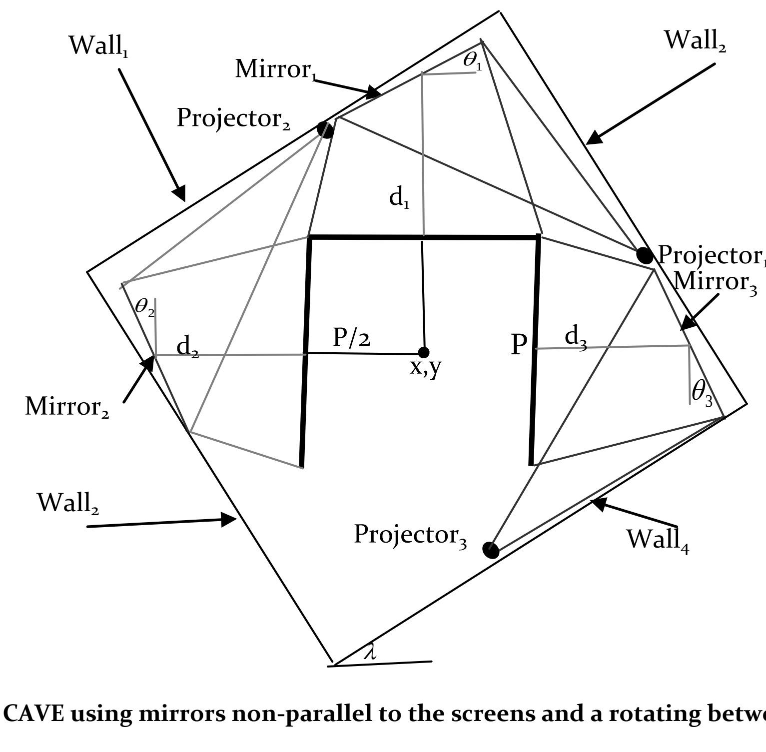


**Figure 4.9 CAVE using mirrors non-parallel to the screens and a rotating between the CAVE and the room walls**

Table 4.2 shows the coordinates of mirrors, the values of $m_1$ and $m_2$ depend on the values of $d_s$ and $\theta_s$, see Eq. 4.11 and Eq. 4.12. Table 4.3 shows the coordinates of the projectors and Table 4.4 shows the coordinates of four lines surrounding all devices. $W\_1_{x,y}$ is the coordinate of the device with a minimum y, $W\_2_{x,y}$ is the coordinate of the device with a minimum x, $W\_3_{x,y}$ is the coordinate of the device with a maximum x, finally $W\_4_{x,y}$ is the coordinate of the device with a maximum y

**Table 4.2 Mirrors coordinates**

| | $X_1$ | Y1 | $X_2$ | $Y_2$ |
|---|---|---|---|---|
| **Eq.4.15** $mirror_1$ | $x-m_1\cos(\theta_1)$ | $y-\frac{p}{2}-d_1-m_1\sin(\theta_1)$ | $x+m_2\cos(\theta_1)$ | $y-\frac{p}{2}-d_1-m_2\sin(\theta_1)$ |
| **Eq.4.16** $mirror_2$ | $x-\frac{p}{2}-d_2+m_1\sin(\theta_2)$ | $y+m_1\cos(\theta_2)$ | $x-\frac{p}{2}-d_2-m_2\sin(\theta_2)$ | $y-m_2\cos(\theta_2)$ |
| **Eq. 4.17** $mirror_3$ | $x+\frac{p}{2}+d_3-m_1\sin(\theta_3)$ | $y+m_1\cos(\theta_3)$ | $x+\frac{p}{2}+d_3+m_2\sin(\theta_3)$ | $y-m_2\cos(\theta_3)$ |

**Table 4.3 Projectors coordinates**

| | X | Y |
|---|---|---|
| **Eq. 4.18** $projector_1$ | $x+(2P-d_1)\cos\left(\frac{3\pi}{2}+2\theta_1\right)$ | $y-\frac{p}{2}-d_1-(2p-d_1)\sin\left(\frac{3\pi}{2}+2\theta_1\right)$ |
| **Eq. 4.19** $projector_2$ | $x-\frac{p}{2}-d_2+(2P-d_2)\sin\left(\frac{\pi}{2}-2\theta_2\right)$ | $y-(2P-d_2)\cos\left(\frac{\pi}{2}-2\theta_2\right)$ |
| **Eq. 4.20** $projector_3$ | $x+\frac{p}{2}+d_3-(2P-d_3)\cos(2\theta_3)$ | $y-(2P-d_3)\sin(2\theta_3)$ |

**Table 4.4 Lines coordinates surrounding all devices**

| | X1 | Y1 | X2 | Y2 |
|---|---|---|---|---|
| **Eq. 4.21** $L_1$ | $w\_1_x-3P\sin(\lambda)$ | $w\_1_y-3P\cos(\lambda)$ | $w\_1_x+3P\sin(\lambda)$ | $w\_1_y+3P\cos(\lambda)$ |
| **Eq. 4.22** $L_2$ | $w\_2_x+3P\cos(\lambda)$ | $w\_2_y-3P\sin(\lambda)$ | $w\_2_x-3P\cos(\lambda)$ | $w\_2_y+3P\sin(\lambda)$ |
| **Eq. 4.23** $L_3$ | $w\_3_x+3P\cos(\lambda)$ | $w\_3_y-3P\sin(\lambda)$ | $w\_3_x-3P\cos(\lambda)$ | $w\_3_y+3P\sin(\lambda)$ |
| **Eq. 4.24** $L_4$ | $w\_4_x-3P\sin(\lambda)$ | $w\_4_y-3P\cos(\lambda)$ | $w\_4_x+3P\sin(\lambda)$ | $w\_4_y+3P\cos(\lambda)$ |

Eq. 4.25 and Eq. 4.26 determine the equation $y=ax+b$ and are used for each line showed in Table 4.4 and combining them with the Eq. 4.27 and Eq. 4.28 the four corners of the room can be computed. With the corners, the length of each wall can be calculated, see Eq. 4.29, and the area (A_4) also, see Eq. 4.30.

**Eq. 4.25** $$a=\frac{y_1-y_2}{x_1-x_2}$$

**Eq. 4.26** $$b=y_1-x_1\frac{y_1-y_2}{x_1-x_2}$$

**Eq. 4.27** $c_x = \frac{b_1 - b_2}{a_1 - a_2}$

**Eq. 4.28** $c_y = \frac{aL_x}{b_1}$

**Eq. 4.29** $wall\_n = \sqrt{(c_{x_1} - c_{x_2})^2 + (c_{y_1} - c_{y_2})^2}$

**Eq. 4.30** $A\_4 = (wall_1)(wall_2)$

Although the equations of this section use radians, the example uses degrees for a better comprehension. In our example, where $P$ = 2.80$m$., using $d_1 = d_2 = d_3 = 1.9$, $\theta_1 = \theta_2 = 26$ and $\theta_3 = 23$, the room size should be of =33.59$m^2$.

## 4.3 System

The equations of this section have been shown using radians, the system only use degrees as input and output parameters, due to degrees produce a better comprehension, but internally the system uses radians.

The equations of the fourth design were validated through a system. Furthermore, the data used were the same of the previous designs., every design use P=2.8 Figure 4.10 shows the CAVE using the configuration of the first design, in this design there are not mirrors, therefore the data of mirror rotation are not used, and the wall rotation is 0°. The horizontal wall length is 8.4m and the length of the vertical wall is 14 m. Finally, the room area is 117.6m², this result is the same using Eq. 4.3

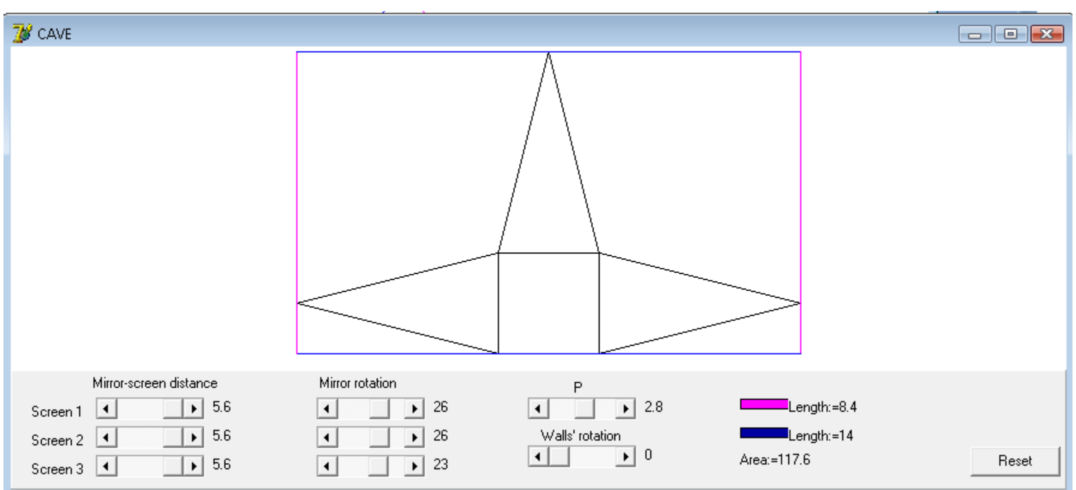


**Figure 4.10 Calculating the area of the design number one.**

Figure 4.11 shows the CAVE using the configuration of the second design, in this design there are not mirrors and the room rotation is 45°. The length of the walls is 9.89 m. The room area is 98m², this result is the same using Eq. 4.4.

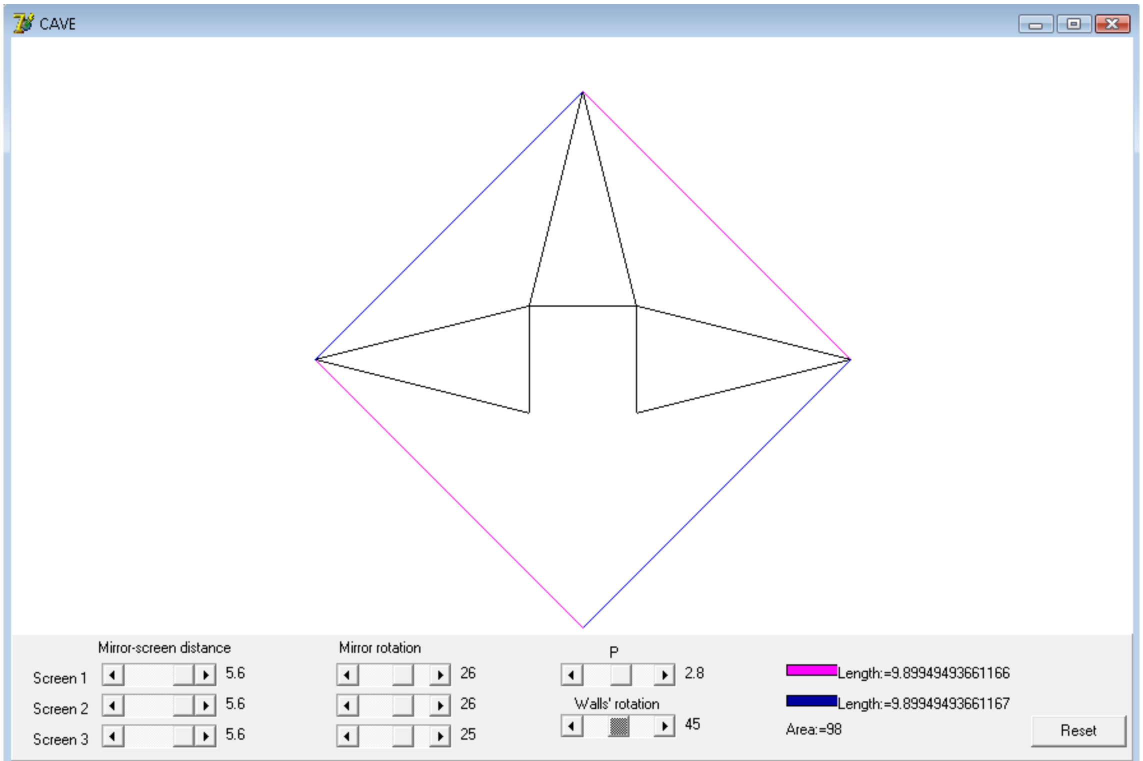


**Figure 4.11 Calculating the area of the design number two.**

Figure 4.12 shows the CAVE using the configuration of the third design, in this design the distances between the mirrors and their respective screen are the same, 2.8m, the mirrors are not rotated and the rotation of the walls is 45°. The length of the walls is 6.92m. The area of the room is 48.02m², the same result is achieved using Eq. 4.5

Figure 4.13 shows the CAVE using the configuration of the fourth design, in this design the distances between the mirrors and their respective screen are the same, 1.9m, two mirrors are rotated 26° and the third mirror is rotated 23°. The rotation of the wall is 32°, the length of the walls is 5.66m and 5.93m. The area of the room is 33.59m².

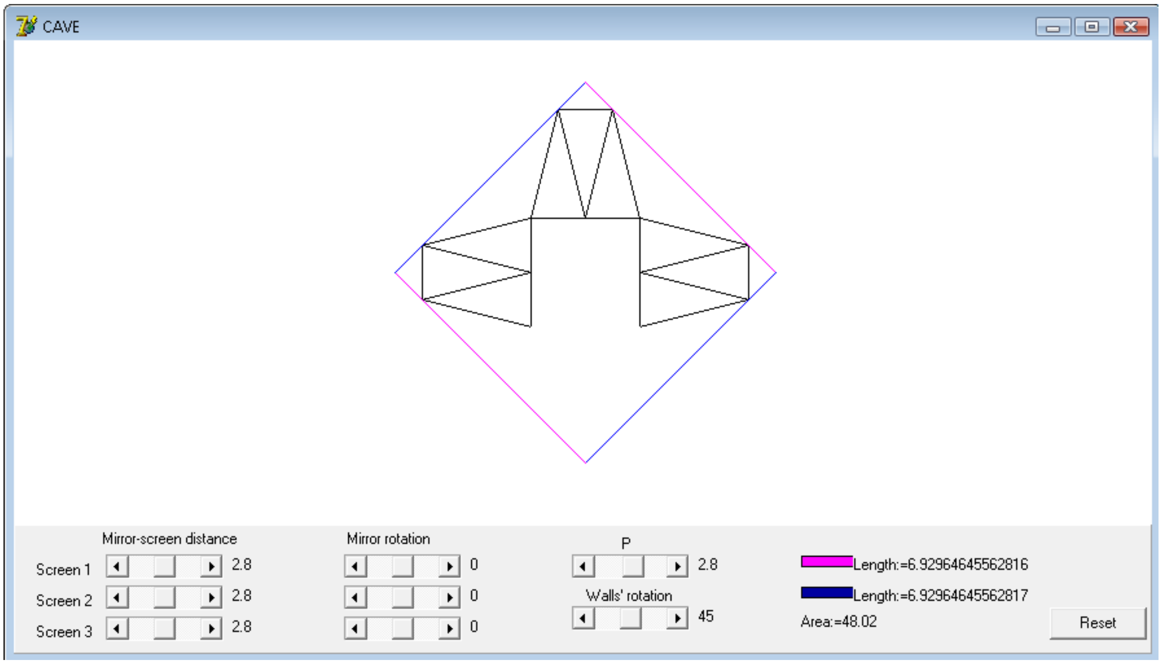


**Figure 4.12 Calculating the area of the design number three.**

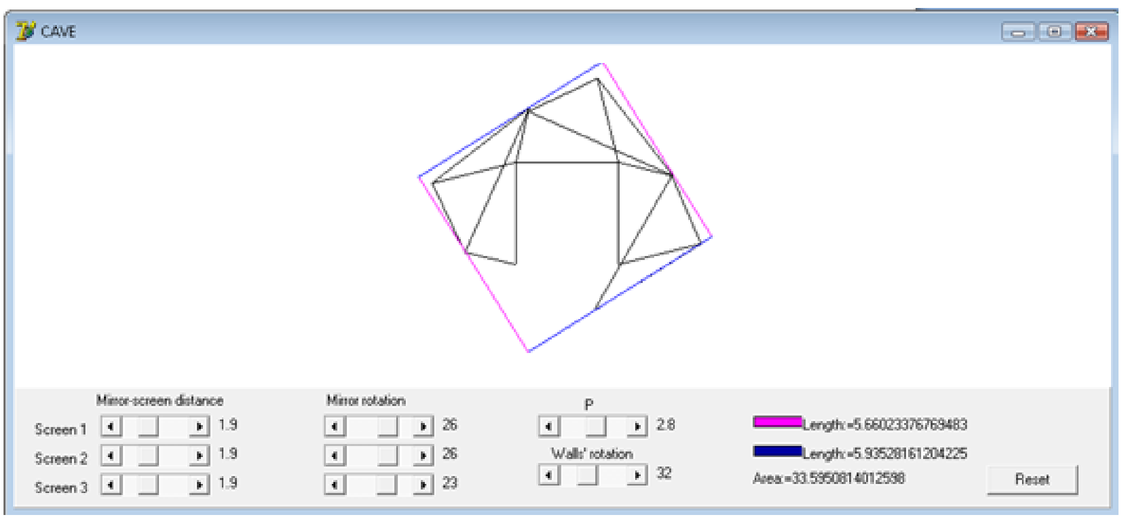


**Figure 4.13 Calculating the area of the design number four.**

## 4.4 Summary

In this chapter four plans about how to set a CAVE have been shown, in addition formulae to determine the space used in each purpose. Three alternatives to reduce the physical requirements were shown, which are: “the technique of the mirror and the projector”, “different ways of distributing the devices involved in the environment” and “rotate some degree the CAVE”.

Therefore, two contributions have been given:

-The development of equations to compute the room size where a CAVE of specific dimensions should be built.

- The development of a system to validate our methodology shown in this section.

Although CAVE models were used in the four designs, the knowledge shown in this chapter can be applied to any environment that uses projectors.

Finally, it is possible to see that the designs 1 and 2 use a great amount of space. The design 3 offers reduction of space because this uses "the technique of the mirror and the projector". The last design is a better option; this design uses the three techniques above-mentioned: "the mirrors and the projector", "a special distribution of the devices and a rotation between the CAVE" and "the room walls".